Now let us go back to (2.23). In view of (2.25) we have

$$A\, e^{\lambda(\xi-\xi_0)} = \frac{e^{-u}}{\left[1-(1-e^{-u})^{\frac{1}{2}}\right]^2} , \tag{2.27}$$

where

$$A = \frac{e^{-u_0}}{\left[1-(1-e^{-u_0})^{\frac{1}{2}}\right]^2} . \tag{2.28}$$

Finally, by means of simple calculatinns, (2.27) allows us to obtain the following expression of u in terms of ξ :

$$u = \ell n \frac{\left[1 + 2Ae^{\lambda(\xi-\xi_0)}\right]^2}{4\, Ae^{\lambda(\xi-\xi_0)}\left[1+e^{\lambda(\xi-\xi_0)}\right]} . \tag{2.29}$$

3. <u>SERIES REPRESENTATION OF</u> $\Psi(\alpha,\beta,\gamma;x)$ <u>IN TERMS OF INCOMPLETE GAMMA FUNCTIONS</u>.

Let us consider the binomial expansion

$$\left(1 - \frac{e^{-t}}{t^{\beta}}\right)^{\gamma} = 1 + \frac{1}{\Gamma(-\gamma)} \sum_{n=1}^{\infty} \frac{\Gamma(n-\gamma)}{n!} \frac{e^{-nt}}{t^{n\beta}} , \tag{3.1}$$

where the series on the right is uniformly convergent for $t \geqslant x$, x being any fixed number such that $e^{-x} < x^{\beta}$.

We can thus write

$$\Psi(\alpha,\beta,\gamma;x) = - \frac{1}{\Gamma(-\gamma)} \sum_{n=1}^{\infty} \frac{\Gamma(n-\gamma)}{n!} \int_x^{\infty} dt\; t^{\alpha-n\beta-1} e^{-nt} . \tag{3.2}$$

Therefore. the use of the following integral representation for the incomplete Γ-function [14]

$$\mu^{\nu}\Gamma(\nu,\mu x) = \int_{x}^{\infty} dt\ t^{\nu-1} e^{-\mu t} ,$$

for $x > 0$ and $\mathrm{Re}\mu>0$, leads us to the expression

$$(3.3)\quad \psi(\alpha,\beta,\gamma;x) = -\frac{1}{\Gamma(-\gamma)} \sum_{n=1}^{\infty} \frac{\Gamma(n-\gamma)}{n!} n^{n\beta-\alpha}\Gamma(\alpha-n\beta,nx).$$

Obviously, in the special case $\gamma = m$, where m is a positive integer, the expression (3.3) reduces to a finite sum of incomplete Γ-functions, specifically:

$$(3.4)\quad \psi(\alpha,\beta,m;x) = -\sum_{n=1}^{m} (-1^{n})\binom{m}{n} n^{n\beta-\alpha}\Gamma(\alpha-n\beta,nx).$$

4. A RECURRENCE RELATION.

The following recurrence relation holds:

$$(4.1)\quad (1-\frac{\gamma\beta}{\alpha})\ \psi(\alpha,\beta,\gamma;x) = -\frac{1}{\alpha} x^{\alpha}\left[1-(1-\frac{e^{-x}}{x^{\beta}})^{\gamma}\right] +$$

$$+\frac{\gamma}{\alpha}\left[\psi(\alpha+1,\beta,\gamma;x) - \psi(\alpha+1,\beta,\gamma-1;x)-\beta\psi(\alpha,\beta,\gamma-1;x)\right] ,$$

for $\alpha \neq 0$.

In fact, from (1.6) we can write

$$\psi(\alpha,\beta,\gamma;x) = \int_{x}^{\infty} dt\ t^{\alpha-1}\left[1-(1-\frac{e^{-t}}{t^{\beta}})(1-\frac{e^{-t}}{t^{\beta}})^{\gamma-1}\right] ,$$

which yields