$$J'E \times_M J'E^* \xrightarrow{<,>} T^*M$$

such that, for each section $u : M \to E$ and $v : M \to E^*$, the following diagram is commutative

$$\begin{array}{ccc} J'E \times_M J'E^* & \xrightarrow{<,>} & T^*M \\ {\scriptstyle (j^1u,\, j^1v)} \nwarrow & & \nearrow {\scriptstyle \Pi^2(j^1<u,v>)} \\ & M & \end{array}$$

Such a map is bilinear $\_$.

14 PROPOSITION.

Let $\eta \equiv (E,p,M)$ be a vector bundle and let $g : E \times_M E \to R$ be a pseudo-Riemannian structure.

There is a unique map

$$\check{g} : J'E \times_M J'E \to T^*M$$

such that the following diagram is commutative, for each section $u,v : M \to E$

$$\begin{array}{ccc} J'E \times_M J'E & \xrightarrow{\check{g}} & T^*M \\ {\scriptstyle (j^1u,\, j'v)} \nwarrow & & \nearrow {\scriptstyle \Pi^2 \circ j^1(g(u,v))} \\ & M & \end{array} \quad .$$

3 LIE DERIVATIVES.

1 DEFINITION.

A K-LIE-DERIVABLE bundle is a 4-plet

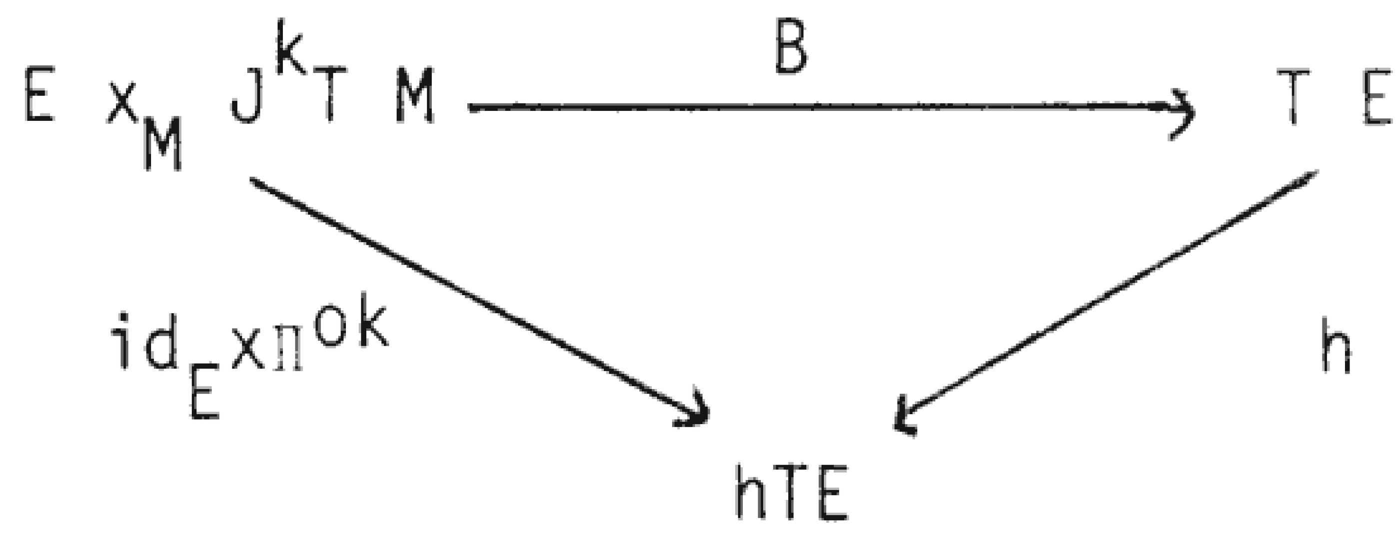
$$\eta \equiv (E,p,M;B),$$

where (E,p,M) is a bundle and

$$B : E \times_M J^kTM \to TE$$

is a bundle morphism on hTE and a linear morphism on E $\_$.

Hence the following diagram is commutative



and B is an affine morphism on hTE.

2 DEFINITION.

Let η be a K-LIE-derivable bundle.

a) The LIE OPERATOR is the map

$$\check{\sim} : J'E \times_M J^k TM \to \bar{\nu}\, TE$$

given by the composition

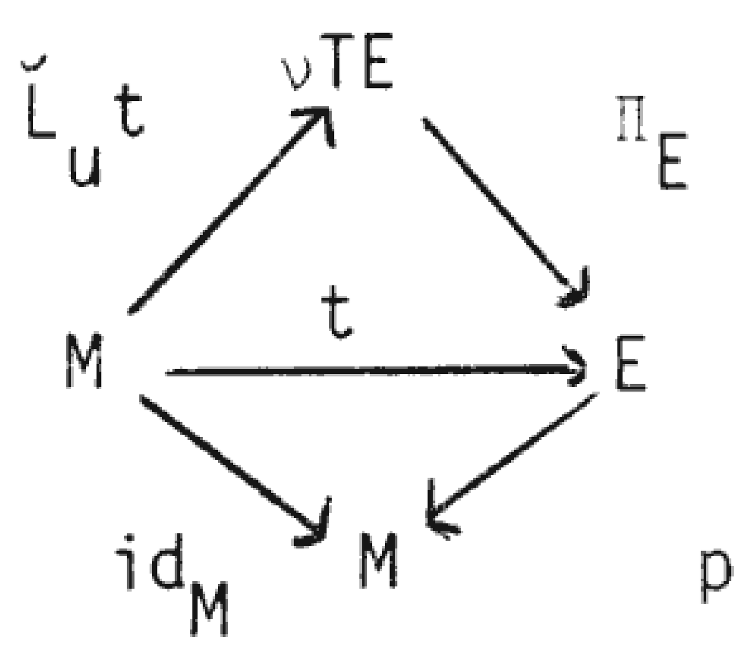
$$J'E \times_M J^k M \to (J'E \times_M TM) \times (E \times_M J^k TM) \xrightarrow{C-B} \bar{\nu}\, TE \quad .$$

b) Let $u : M \to TM$ and $t : M \to E$ be sections.

The LIE DERIVATIVE of t with respect to u is the section

$$\check{L}_u t \equiv (j^1 t, j^k u) : u \to \bar{\nu}\, TE \; .$$

Hence the following diagram is commutative



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3 PROPOSITION.

We have

a) $$\check{L}_{(u+u')} v = \check{L}_u + \check{L}_u v$$

b) $$\check{L}_{fu} v = f\, \check{L}_u v - B(v, t \circ (j^k f \otimes j^{k-1} u))$$ $\dot{-}$

4 If η is a vector bundle, we denote by

$$\check{\mathcal{L}} : J'E \times_M J^kTM \to E$$

the map

$$\mathcal{L} \equiv \perp\!\!\perp_E \circ \check{\mathcal{L}}$$

and by

$$L_u t : M \to E$$

the map

$$L_u t \equiv \perp\!\!\perp_E \circ \check{L}_u t \;.$$

5 PROPOSITION.

Let η be a vector bundle and let B be a linear morphism on $J^kTM \to TM$

Then we have

$$L_u(t+t') = L_u t + L_u t'$$

$$L_u(ft) = f\, L_u t + (u.f)t \quad \dot{-}$$

6 PROPOSITION.

Let η' and η'' be vector bundles and let B' and B'' be linear morphisms on $J^kTM \to TM$.

Then there is a unique linear morphisms on $J^kTM \to TM$

$$B : (E' \otimes_M E'') \times_M J^kTM \to T(E' \otimes_M E'')$$

such that the following diagram is commutative

$$\begin{array}{ccc}
E' \times_M E^k \times_M J^kTM & \longrightarrow & (E' \times_M J^kTM) \times (E'' \times_M J^kTM) \\
 & & \downarrow B' \times B'' \\
\downarrow & & TE' \times_{TM} TE'' \\
 & & \downarrow t \\
(E' \otimes_M E'') \times J^kTM & \xrightarrow{\;B\;} & T(E' \otimes_M E'')
\end{array}$$

Then $(E' \otimes_M E'', p, M; B)$ results into a Lie derivable bundle.

Furthermore, we get

$$L_u(t' \otimes t') = L_u t' \otimes t'' + t' \otimes L_u t'' \quad \dot{-}$$

7 EXAMPLE.

Let $\eta \equiv (E,p,M;B)$ be a 0-Lie derivable bundle.

Then $$B : E \times_M TM \to TE$$

results into a horizontal section (see [7], §5).

Moreover, if η is a vector bundle and B is a linear morphism on TM, the 0-Lie-derivative coincide with the covariant derivative.

8 EXAMPLE

We get the usual Lie derivative of tensors $M \to T_{(p,q)}M$, taking into account the previous proposition and the 1-Lie-derivable bundles

$$\eta \equiv (TM, \Pi_M, M; s \circ c) \quad \text{and} \quad \eta \equiv (T^*M, \rho_M, M, C^*) \quad .$$

9 EXAMPLE.

Let $\beta \equiv (E,p,M;B)$ a bundle of geometric objects (see [7]) .

Let β be of "order K", i.e. such that the following condition holds: if $v \in J^kTM$, $x', x'' : M \to TM$ are two representative of v and f', f'' are the one parameter groups generated by x', x''

then $$\partial(Bf') = \partial(Bf'') \quad .$$

Then the map
$$B : E \times J^kTM \to TE,$$
given by $$B(e,v) \equiv \partial(Bf)(e)$$

makes $(E,p,M;B)$ a k-Lie derivable bundle.

4 CONNECTION ON A BUNDLE.

Let $\eta \equiv (E,p,M)$ be a bundle.

1 DEFINITION.