10. CONSTRUCTION OF SOME LINEAR SYSTEMS

LEMMA 10.1: Let |D| be a complete, non-special linear system and let j_0, \ldots, j_n be the (|D|, P)-orders, where n=dim|D|. Then the (|D+P|, P)-orders are 0, $j_0 + 1, \ldots, j_n + 1$.

THEOREM 10.2: If |D| is a complete, non-special, classical, linear system and |D'| is a complete, base-point-free, linear system, then |D+D'| is classical.

Let Pe% and let j_0, \ldots, j_n be the (\mathcal{D}, P)-orders for \mathcal{D} canonical. Then $j_0+1=\alpha_1, \ldots, j_{g-1}+1=\alpha_g$ are the <u>Weierstrass gaps</u> at P; that is, there does not exist f in $\bar{K}(%)$, regular outside P, such that $\operatorname{ord}_P(f)=-\alpha_i$.

THEOREM 10.3: Let PeC and let $\alpha_1, \ldots, \alpha_g$ be the Weierstrass

gap sequence at P. If the linear system $\mathscr{D} = |dP|$ for some positive integer d, then the (\mathscr{D}, P) -orders are $\{0, 1, \ldots, d\} \setminus \{d - \alpha_i \mid \alpha_i \leq d\}$. THEOREM 10.4: With P and $\alpha_1, \ldots, \alpha_g$ as above, let V be a canonical divisor, $s \geq 2$ an integer, and $\mathscr{D} = |V+sP|$. Then the (\mathscr{D}, P) -orders are

$$i = i$$
 for $i=0,1,...,s-2$,
 $s-2 = s-1+\alpha$ for $i = 1,...,g$.

THEOREM 10.5: Let P in \mathscr{C} be an ordinary point for the canonical linear system |V| and assume that |V| is classical. Then, for any n such that $0 \leq n \leq g-1$, the linear system $\mathscr{D} = |V-nP|$ is a classical γ_{2g-2-n}^{g-1-n} without base points, and P is \mathscr{D} -ordinary.

An important result an linear series is also worth noting.

THEOREM 10.6: The generic curve of genus g has a γ^n_d if and only if

$$d \geq \frac{n}{n+1} g+n$$
.

11. THE ESSENTIAL CONSTRUCTION

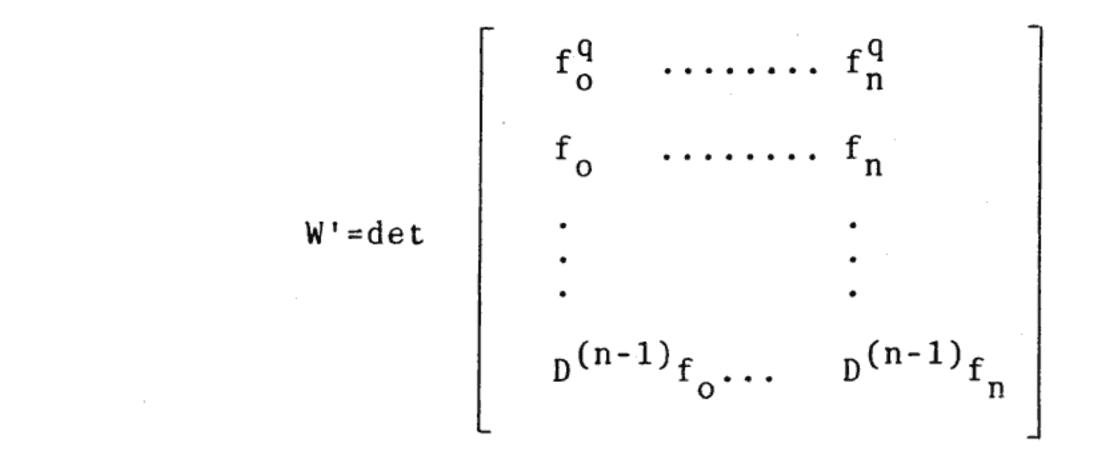
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Given the curve & with its linear system of hyperplanes and with N the number of its GF(q)-rational points, consider the set $\mathscr{F} = \{P | P \varphi c H_p\}$; compare §4 for the plane. So $P \epsilon \mathscr{F} \iff$

$$det \begin{bmatrix} f_0^q & \dots & f_n^q \\ \begin{pmatrix} (j_0) \\ D_t & f_0 & \dots & D_t & f_n \\ \vdots & \vdots & \vdots & \vdots \end{bmatrix} = 0$$

$$\begin{bmatrix} \binom{j_{n-1}}{D_t} & \binom{j_{n-1}}{f_0} \\ 0 & t \end{bmatrix}$$

To give an outline first, take the classical case in which $j_i = i$. So, let



If $W' \neq 0$, then W is a function of degree