

# PREFACE

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The main subject of this work is the analysis of the asymptotic behaviour of the solution of the Cauchy problem:

$$u'(t) = Au(t) \text{ with } u(0) = x \in D(A),$$

where  $A$  generates a  $C_0$ -semigroup on a Banach space  $E$ . We are particularly interested in *positive*  $C_0$ -semigroups on Banach *lattices*.

The theory of generation of semigroups of linear contractions, which is at the basis of a theory of evolution equations, was developed by Hille and Yosida in 1948. W. Feller (1952) and R.S. Phillips (1962) obtained first results concerning the characterization of the generators of special positive semigroups.

On the other hand, in the 60's and 70's the theory of ordered Banach spaces and positive operators was developed and is well documented in the monographs by H.H. Schaefer [26], A.C. Zaanen [37], Aliprantis and Burkinshaw [1], Meyer-Nirberg [21] and many others.

In the 80's, applications of positivity to Cauchy problems and specially to concrete evolution equations from transport theory, mathematical biology, and physics, has attracted much interest and was the subject of many papers. Most results of what was known around 1985 about this subject can be found in the book written by the functional analysis group in Tübingen, see [22]. This led to remarkable progress during the last decade.

We have organized these notes as follows.

We concentrate our attention on the asymptotic behaviour of positive  $C_0$ -semigroups of linear operators on Banach lattices and applications to transport theory.

In Chapter 1 we recall some basic and useful results on Banach lattices and positive operators. In Chapter 2 we discuss the uniform exponential stability of  $C_0$ -semigroups and present the Perron-Frobenius theory and its application to the

asymptotic behaviour of irreducible  $C_0$ -semigroups. The last Chapter is dedicated to the application of our results to transport equations.

We have assumed that the reader is already familiar with basic functional analysis and the theory of  $C_0$ -semigroups on Banach spaces.

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