CAPITOLO 4

CONVEGNI

4.1 CONVEGNO IN MEMORIA DI E. DE GIORGI
S.N.S., PISA, 20–23 OTTOBRE 1997

4.1.1 Discorso introduttivo al Convegno di *G. Letta*

Gli amici che hanno organizzato questo convegno hanno voluto che io dicessi a questo punto due parole come vicedirettore del Dipartimento di Matematica dell' Università di Pisa.

Io li ringrazio di questo invito, che mi permette di portare qui, in modo ufficiale, il saluto augurale del mio Dipartimento e di complimentarmi con loro per la bella iniziativa di organizzare questo convegno in onore di Ennio De Giorgi.

Ma dico subito che trovo un poco innaturale questa mia veste "ufficiale", e sono sicuro che la troverebbe innaturale lo stesso Ennio, che sentiva come pochi l' unità profonda della comunità matematica pisana, al di là e al di sopra di tutte le barriere, divisioni o articolazioni di carattere amministrativo.

E dunque preferirei, col permesso di chi mi ha invitato, considerarmi qui semplicemente come uno che ha condiviso con molti degli amici presenti la fortuna di essere allievo e amico di De Giorgi, e che ora si sente orfano di lui.

Ho detto "allievo", e forse a stretto rigore il termine non è tecnicamente esatto, dal momento che ho scritto con lui un solo lavoro. Ma quale dei matematici pisani non può dirsi, direttamente o indirettamente, in misura maggiore o minore, allievo di Ennio, e quindi suo orfano?

Ennio era il porto sicuro per ciascuno di noi. Ci sovrastava con la sua eccezionale statura scientifica, intellettuale, umana. Ma, lungi dall' essere, come normalmente avviene nel caso di scienziati di altissimo livello, geloso amministratore del proprio tempo, era, al contrario, sempre disposto a concedere ascolto a chiunque glielo chiedesse.

Qualunque fosse il livello dell' interlocutore o della questione a lui posta, egli accettava l' incontro con spontaneo, genuino interesse: sempre animato da una curiosità vivissima, da una semplicità autentica, che non aveva nulla di artefatto, che non era falsa modestia o umiltà costruita, e che era sorretta da un candore quasi di fanciullo, unito a una prodigiosa forza intellettuale.

E in questi incontri il vero miracolo (non saprei come altrimenti definirlo) era che l' interlocutore non si sentiva mai schiacciato o paralizzato dalla personalità scientifica che gli stava di fronte, ma al contrario si trovava subito a suo agio, come se parlasse con un suo familiare.

Non spetta a me qui ricordare la folgorante avventura scientifica di Ennio De Giorgi: altri, ben più qualificati di me, ne illustreranno le varie tappe con dovizia di particolari.

Mi limiterò ad osservare che la figura di De Giorgi sfugge ad ogni tentativo di facile catalogazione.

Si può dire di lui che è stato uno scienziato di eccezionale creatività, che ha lasciato nella matematica una traccia profonda e duratura: e certamente non si sbaglia.

Si può aggiungere che ha concepito la ricerca matematica, e più in generale la ricerca scientifica, come una parte di ciò che egli amava chiamare, con espressione biblica, "amore della Sapienza".

Si può dire ancora che ha avuto molto a cuore, e ha promosso con grande efficacia, la difesa dei diritti umani, vista come corollario di un principio più alto: la fede nella dignità della persona umana.

Si può dire infine che non ha conosciuto sentimenti pur tanto diffusi tra gli uomini di scienza (anche sommi), quali l' arroganza del successo o il desiderio del potere, e che, al contrario, è stato un uomo profondamente buono, solo in apparenza indolente, in realtà animato da una costante tensione intellettuale e morale, e sorretto da una visione religiosa della vita che lo rendeva capace di infondere in coloro che gli erano accanto serenità, fiducia, coraggio.

Ma tutte queste parole non bastano a dare neppure una pallida idea del la sua irripetibile personalità a coloro che non abbiano avuto la fortuna di conoscerlo.

Più utile sarà forse, per terminare, citare alcune parole dello stesso De Giorgi, che io considero particolarmente illuminanti:

> *Il buon "servo della Sapienza" riconosce onestamente i limiti della propria intelligenza e della propria cultura, svolge con modestia e pazienza il proprio lavoro quotidiano, ma non esclude l' eventualità che la stessa Sapienza gli venga incontro con una coincidenza inattesa, un' osservazione fortunata, un' intuizione felice.*

E a queste splendide parole vorrei aggiungere che di siffatti incontri fortunati con la Sapienza è mirabilmente costellato, dal principio alla fine, tutto il lungo cammino scientifico di Ennio De Giorgi, tanto da legittimare il sospetto che la Sapienza avesse stretto, con lui, un patto del tutto speciale.

Non mi rimane dunque che da formulare l' augurio che al presente convegno, concepito nel suo nome e in sua memoria, possano estendersi gli effetti benefici di quello speciale patto tra lui e la Sapienza.

4.1.2 Discorso di *W. Fleming*

Geometric measure theory.

During the 1950s and 1960s both De Giorgi and I were working in what is now called geometric measure theory. These remembrances concern mostly some memories of De Giorgi and his brilliant work during that time period. Geometric measure theory provides class of objects, which I will call in an imprecise way "surfaces" of arbitrary dimension k in some euclidean space. They were called "generalized surfaces" by L. C. Young, "varifolds" by F. Almgren and "integral currents" by H. Federer and myself.

For De Giorgi, the objects were portions of the reduced boundary of a set of finite perimeter, in codimension 1, and later a particular class of what he called "correnti quasinormali" in arbitrary codimension. Of course, the objects are not really smooth surfaces in a classical sense, but it happens that they coincide approximately (in a suitable measure theoretic sense) with finite unions of surfaces of class C^1. The theory provides compactness of sequences of surfaces with bounded k dimensional area and boundaries with bounded $(k-1)$–dimensional area. Another important property is that versions of the classical theorems of Gauss–Green and Stokes remain true.

Geometric multidimensional problems of the calculus of variations provided an important motivation for geometric measure theory A famous example is the Plateau problem, which is to find a k–dimensional surface with least k–dimensional area, among all surfaces with the same boundary. Geometric measure theory provides immediately the existence of an area minimizing surface. However, the problem of regularity of area minimizing surfaces turned out to be quite complicated. The most which can be expected is regualarity except at points of some lower dimensional singular set. In codimension 1, the singular set is empty in low dimensions. However, the famous 1969 Bombieri–De Giorgi–Giusti paper (which will be mentioned again later) shows that this is false in higher dimensions.

Sets of finite perimeter.

I first heard about De Giorgi in 1956 or 1957 when the French mathematician C. Pauc urged me to read De Giorgi's important new papers in the Annali di Matematica and Richerche di Matematica, on sets of finite perimeter (also called at that time Caccioppoli sets). From the Annali paper I first learned about the “slicing formula” which equates the total gradient variation of a function and an integral of the areas of level sets. This formula was used by De Giorgi to show that his definition of set of finite perimeter was equivalent to another definition of Caccioppoli. The slicing formula anticipated the so–called coarea formula, of which it is a particular case.

First main regularity theorem.

In 1961 De Giorgi published two seminal papers in a Seminario di Matematica della Scuola Normale Superiore di Pisa series, which was not I think widely available. This work provided the first big regularity result for the Plateau problem in codimension 1. The proof of this result is an amazing “tour de force”. Starting with a locally area minimizing surface, which is not even known to be locally the graph of a function, De Giorgi managed to prove that the surface is smooth near any point at which it is measure theoretically close to some approximate tangent plane.

Genova workshop.

In August 1962 J. P. Cecconi hosted a workshop at the Università di Genova, at which I first met De Giorgi. In addition to several other Italian mathematicians, E. Reifenberg, from England also attended. Reifenberg had recently written an important 1960 Acta Mathematica paper on the Plateau problem. This workshop had a fundamental role in stimulating further work in geometric measure theory. As Reifenberg said, it was conducted in a kind of “lingua mista”. Despite some language difficulties, many interesting ideas were circulated and taken home for further study.

Visit to the USA.

In 1964 De Giorgi visited Brown and Stanford universities. He came by ship (the Leonardo da Vinci), and I met him in New York. There was a delay of several hours waiting for the passengers to disembark, because of a dock workers strike. During the auto trip from New York to Providence, De Giorgi told me that he had just proved a striking result called the Bernstein theorem for minimal surfaces of dimension 3 in 4 dimensional space. However, there was no mathematics library on the Leonardo da Vinci, and he wished to be certain about the strong maximum principle for elliptic PDEs which he needed in the proof. I assured him that what he needed is OK. We will return to the Bernstein problem in a moment.

During his stay at Brown, De Giorgi gave a series of lectures on what he called “correnti quasinormali”. His approach provided an alternative to the one taken by Federer and myself for normal currents. De Giorgi’s method has the advantage that no use was made of a difficult measure theoretic covering theorem of Besicovitch.

Minimal cones and the Bernstein problem.

In 1969, Bombieri, De Giorgi and Giusti published a truly remarkable paper on area minimizing cones and the Bernstein problem. The results were unexpected, and at least for some analysts contrary to intuition. Speaking only of the Bernstein problem, the question is as follows. Let f be a smooth function of m variables which satisfies the minimal surface PDE in all of m–dimensional euclidean space. Must f be a linear function? This was known for a long time to be true if $m = 2$. It was proved by geometric measure theory methods by De Giorgi for $m = 3$, then by Almgren for $m = 4$ and by J. Simons for $m = 5, 6, 7$. However, Bombieri, De Giorgi and Giusti showed that the result is false for $m \geq 8$.

I was visiting at Stanford during 1968–69, when this startling news arrived. At least one of the analysts there was rather upset, asking "How can a theorem in analysis be true for functions of 7 or fewer variables but not for functions of 8 variables?" After some reflection he achieved peace concerning the matter with the wise observation that the Bernstein problem really belongs to geometry rather than to analysis.

Further remarks.

After the 1960s De Giorgi's work and mine took different directions. However, we kept up a lifelong friendship and saw each other from time to time, both in Pisa and elsewhere. Communication became easier as De Giorgi's English improved and I learned a little Italian. (The other choice was bad French which we mutually decided against early on.) Besides his mathematical work, De Giorgi told me about his trips to Eritrea and his work for Amnesty International. Our last meeting was in 1993 at the 75th birthday conference for Cecconi in Nervi.

Ennio De Giorgi was a mathematician of extraordinary depth and powerful insights. There is a great Italian tradition in the calculus of variations, and among the world leaders in the first part of the 20th century was L. Tonelli. De Giorgi was in every sense a worthy successor to Tonelli. There is a plaque on a wall in the old Università di Pisa building complex concerning Tonelli. While I don't remember the exact wording, it says in effect that Tonelli was both an excellent mathematician and outstanding citizen. The same can be said about De Giorgi, although his good citizenship was shown perhaps in a different style from Tonelli's.

We miss him very much.

4.1.3 Discorso di *E. Vesentini*

I consider it a privilege having been asked to say a few words at the opening of this conference and for having thus been given the opportunity to welcome Ennio De Giorgi's relatives and the many friends and colleagues who gathered here today to honor his memory.

Ennio left us one year ago, but, fittingly, his office in this building has been kept untouched until recently, and it is as he will be back shortly and resume once more his duties at the beginning of a new academic year.

My friends asked me to take part in this opening session only a few days ago, but — even if I were allowed more time — I could not, in any way, prepare something that might look as a systematic account of De Giorgi's mathematical achievements. I can only offer a personal recollection of a friendship that lasted more than forty years.

I am a mathematician and I have been Ennio's colleague, here in Pisa, since 1961, but my scientific interests were quite removed from those of De Giorgi, and I did not have the chance of doing research with him. When we discussed technical mathematics — and that occurred with some regularity when Aldo Andreotti and I were working together on problems of potential theory on complex manifolds. Aldo and I were always on the receiving end: the two of us asking questions and De Giorgi answering in his typical, unassuming, nonchalant way, suggesting answers and approaches that, at first sight, seemed sometime only vaguely related to our original questions. Only afterwards we discovered, more often than not, that he had clearly perceived the real difficulty behind our problems and his answers were correctly focused on something concrete.

We first met in 1951 in Rome, both in our early twenties. We were part of a group of young research fellows in the University of Rome, at the Istituto per le Applicazioni del Calcolo, at the Istituto Nazionale di Alta Matematica: Caligo, Pucci, Capriz, Sce, Bertolini, Aparo, . . . Ennio was an active member of that group, but, going back with my memory to those remote and happy days, I remember him as somewhat insulated from the rest of us. Mathematics is a pervasive activity and has a natural, strong tendency to creep into many apparently remote aspects of daily life. That was particularly true for Ennio, who could stop abruptly whatever might be his temporary occupation and start scribbling garbled mathematical symbols on any peace of paper he could lay his hands on. One could feel — and I felt — that his thoughts were travelling on a different wave–length. Thinking back, I realize that, paradoxically, at that lime his foresight was — in some sense — too farsighted.

I will only mention one episode. In the early fifties, the geometry of differentiable manifolds was booming, in the hands of G. de Rham, K. Kodaira, W. Hodge, H. Hopf. H. Cartan, A. Lichnrowicz, . . . I was studying the theory of characteristic classes on complex manifolds, while Ennio was working on what was known among us as the "ship problem". As an outgrowth of his work, he was trying to introduce new measures fitting the various requirements posed by his original problem. One of the preliminary questions was that of finding a framework for the measure; that is to say, finding a suitable definition of the space carrying the measure and keeping the definition as supple and general as possible. A natural first candidate was obviously a finite dimensional manifold with a higher or lower class of differentiability. Ennio, who notoriously did not like visiting mathematical libraries, asked me to give him a crash course of what we call now differential topology. I did my best, but, at the end of my sketchy description of the most relevant geometric objects related to manifolds (tangent vectors, differential forms, currents, . . .), he felt that the measure–theoretic implication of the space being differentiably locally euclidean was too stringent. Thus, he discarded differentiable manifolds as too regular, saying: "Your manifold is locally a ball; my manifold should behave locally as a sponge".

I heard the same word in Torino last year. Dennis Sullivan was delivering the Guido Fubini Lectures on The foundations of geometry, analysis and the differentiable structure of manifolds. In one of his lectures he offered a new, original, exciting reading of Riemann's Habilitationschrift; über die Hypothesen welche die Geometrie zu Grunde liegen. After reviewing the ways of gauging infinitesimal vicinity and classifying different possibilities related respectively to phase signature operators and locally euclidean conformal gauges, signature operators and locally euclidean metric gauges, Dirac operators and locally euclidean differentiable manifold, D. Sullivan concluded foreseeing a manifold looking locally as a sponge. A similar vision to the one De Giorgi had more than forty years before, previous to any substantial work carried out by Sullivan himself, Donaldson, Teleman. Connes, and certainly without any direct knowledge of Riemann's Habilitation Schrift.

After those years in Rome, our careers followed parallel patterns. We won the national competitions for full professorships at the same time, at the end of 1958. I was appointed Professor of Geometry in the University of Pisa, in February 1959. Ennio became Professor of Mathematical Analysis in Messina, in November 1958, and in 1960 moved to Pisa, as Professor of Mathematical Analysis in the Scuola Normale Superiore. In those years, a group of young mathematicians gathered in Pisa, on invitation of Sandro Faedo, who at that time was Rector of the University: Andreotti, De Giorgi, Barsotti, Stampacchia, Prodi and, a little later, Bombieri. Some good mathematics was produced in Pisa in those years, but that is a different story, which has been told already.

In 1961, Andre Weil visited Pisa. The article Sulla differenziabilità e l' analiticità degli estremali degli integrali multipli regolari, that Ennio De Giorgi had published a few years before in the "Memorie dell' Accademia delle Scienze di Torino" had caught the attention of Louis Niremberg and other mathematicians in the United States, in spite of the fact that it was written in italian and had appeared in a Journal not easy to find in mathematical libraries. Weil, well aware of the importance of the result and perhaps guessing already its potential relationship with the solution of the nineteenth problem of Hilbert, tried to convince Ennio to spend some time at the Institute for Advanced Study in Princeton. He did not succeed, and I still have a letter by Weil, where he comments on Ennio's stubborness. However, two years later De Giorgi accepted an invitation to visit the United States, and spent a few weeks of the Winter of 1964 in Providence, R. I., at Brown University. Providence is not far from Cambridge, Mass., where I was spending the academic year 1963-64 with my family, and Ennio came several times to visit us. His visits happened to coincide often with winter-storms in the area, and our little son Massimo saw Ennio has a sort of a friendly snowman appearing amid flurries of snowflakes.

Toward the end of his séjour in the United States, Ennio was invited by Irvin Segal to give a lecture on the Bernstein Problem. I must confess that, while I was driving him to MIT on the day of his talk, I was a little worried. As Bombieri has recalled in his eulogy of De Giorgi in the Accademia Nazionale dei Lincei, Ennio had a fantastic geometric imagination. On the other hand, specific mathematical formulas looked some time a sort of encumbrance to him, whereas his verbal exposition happened to be often crystal clear. But achieving this quality requires mastering the language, and Ennio's english was poor. Well, I was mistaken in worrying. The lecture turned out to be, one of the very good ones I ever listened to, and, in absolute, one of the best delivered until then by Ennio De Giorgi; sentences neatly stated and clearly written on a well organized blackboard, proofs elegantly balanced on their essential points, without redundancies. The reason of the success was, paradoxically, in Ennio's poor knowledge of the language: the fact that it could master only a limited number of english words forced him to spare and cast them exactly in the right spot.

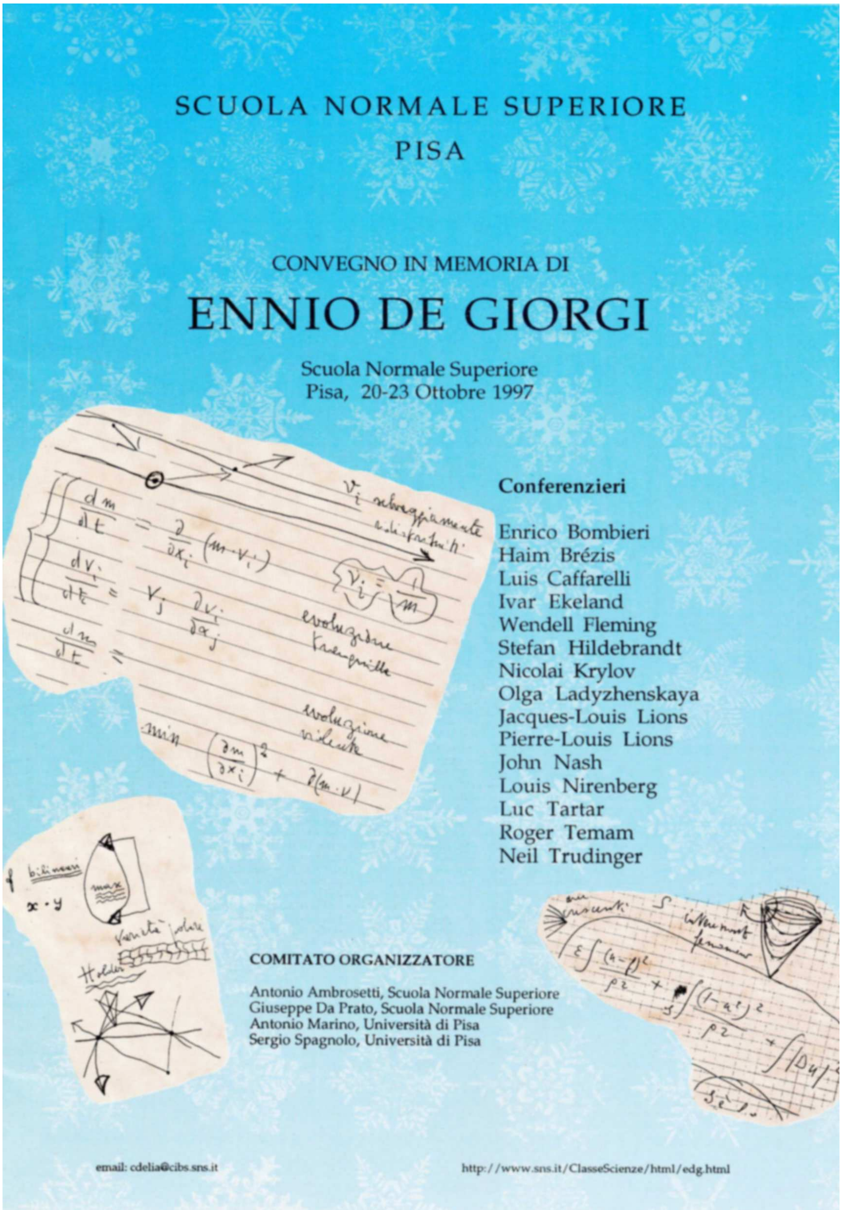
Ennio's teaching was not always so well organized, and it was more effective when addressed to a small group of selected mathematicians. But certainly this fact did not arise from an aristocratic attitude. The clarity of teaching, the necessity of being understood was one of his main worries. If someone in the audience, a student, a colleague, did not follow his arguments, he apologized as if somehow he had failed his duties as a teacher, and was always ready to start again from the beginning.

Coming back to Pisa from Cambridge at the end of 1964, I found Ennio, and together with Luigi Radicati, we resumed our partnership in the entrance examinations to the Scuola Normale Superiore; a demanding job that Ennio discharged with great attention. He was extremely generous with young people when they were in the Scuola, to the point that any decision to expel someone was always taken — when it had to be taken — against his strong opposition. But the admission examination was a completely different story. Grading the candidates was something that he considered a duty to be faced with extreme attention and impartiality. But he believed that a student, ounce admitted to the Scuola Normale, should be given the possibility to develop according to his own intellectual metabolism. The large number of good mathematicians that grew up under his direction — many of whom are present here today — indicate that, ounce more, he was right.

Well, looking from outside, that is all one could say about Ennio De

Giorgi: a great mathematician, a good teacher, a dear friend, a good man. However, the simplicity of his life, his unobtrusiveness should not let us forget the many academic signs of recognition he received in his life: member of the Accademia Nazionale dei Lincei, of the Accademia dei Quaranta, of the Istituto Lombardo di Scienze e Lettere, of the Accademia delle Scienze di Torino, of the Pontificia Academia Scientiarum, foreign member of the Académie des Sciences and of the National Academy of Sciences of the United States (the first italian mathematician after Vito Volterra); Wolf Prize for Mathematics, Premio del Presidente della Repubblica. Laurea Honoris Causa of the Université de Paris, ...

That is all I have to say, thanking again the organizers of this Conference for having given me the opportunity of bringing back the memory of a personal friendship which lasted for more than forty years and — together with that with Aldo Andreotti and Guido Stampacchia — graced those years and my life.


SCUOLA NORMALE SUPERIORE
PISA
CONVEGNO IN MEMORIA DI
ENNIO DE GIORGI
Scuola Normale Superiore
Pisa, 20-23 Ottobre 1997
Conferenzieri
Enrico Bombieri
Haim Brézis
Luis Caffarelli
Ivar Ekeland
Wendell Fleming
Stefan Hildebrandt
Nicolai Krylov
Olga Ladyzhenskaya
Jacques-Louis Lions
Pierre-Louis Lions
John Nash
Louis Nirenberg
Luc Tartar
Roger Temam
Neil Trudinger
COMITATO ORGANIZZATORE
Antonio Ambrosetti, Scuola Normale Superiore
Giuseppe Da Prato, Scuola Normale Superiore
Antonio Marino, Università di Pisa
Sergio Spagnolo, Università di Pisa
email: cdelia@cibs.sns.it
http://www.sns.it/ClasseScienze/html/edg.html

Lunedì 20 ottobre

9.00 **Prolusione**

10.30 COFFEE BREAK

11.00 **Louis Nirenberg** (Courant Institute)
Some properties of solutions of elliptic equations

12.00 **John Nash** (Princeton University)
On the possibility of an "additive metric theory of gravitation" and space-time

13.00 LUNCH

16.00 **Nicolai Krylov** (University of Minnesota)
Hölder continuity and L^p estimates for elliptic equations under general Hörmander conditions

17.00 COFFEE BREAK

17.30 **Roger Temam** (Université Paris XI),
Long time behaviour of solutions of weakly dissipative equations

Martedì 21 ottobre

10.00 **Haim Brézis** (Université Paris VI)
Lifting, traces and asymptotics for the Ginzburg-Landau functional

11.00 COFFEE BREAK

11.30 **Pierre-Louis Lions** (Université Paris IX)
On ordinary and partial differential equations

13.00 LUNCH

16.00 **Jacques-Louis Lions** (Collège de France)
Remarks on controllability and stiffness

17.00 COFFEE BREAK

17.30 **Neil Trudinger** (Australian National University)
Hessian operators, integrals and measures

Mercoledì 22 ottobre

10.00 **Stefan Hildebrandt** (Universität Bonn)
On minimal surfaces with free boundaries

11.00 COFFEE BREAK

11.30 **Luis Caffarelli** (Courant Institute)
Some issues involving the Monge Ampère equation

13.00 LUNCH

16.00 **Wendell Fleming** (Brown University)
Deterministic nonlinear filtering

17.00 COFFEE BREAK

17.30 **Luc Tartar** (Carnegie-Mellon University)
Homogenization of first order equations

Giovedì 23 ottobre

10.00 **Ivar Ekeland** (Université Paris IX)
Systems of PDE's arising in economic theory

11.00 COFFEE BREAK

11.30 **Ol'ga Ladyzhenskaya** (Russian Academy of Sciences)
On applications of the reverse Hölder inequalities to some hydrodynamical problems

13.00 LUNCH