1 Introduction

The results achieved in recent years by the theory of product differentiation may well explain its increasing relevance in the analysis of industrial organization and in the study of the sources of market power. The theory rests on the idea that in the presence of a differentiated demand, the strategic interaction among firms develops along two lines: the prices charged and the characteristics chosen by a firm and its competitors. One of the most investigated topics in this field is the analysis of locational equilibria in a horizontally differentiated market. The horizontal Hotelling model has been widely used in order to discuss problems related to the spatial price competition, the optimal product attributes, the optimal plant location, etc., and has found applications in the spatial economics literature, as well as in trade and banking theory. These models primarily focus on the existence of a Principle of Maximal or Minimum Differentiation (Economides 1986). This existence problem amounts to asking whether the interplay between the structure of consumers' preferences for the differentiated product and the optimal strategic behaviour of firms results into too little or too much product diversity.

Following Hotelling (1929), D'Aspremont, Gabszewicz and Thisse (1979) established a Principle of Maximal Differentiation, by assuming that the intensity of consumer preferences for their ideal product may be reformulated in a locational setup in terms of quadratic transportation costs: in a duopoly market, the firms try to set up apart from each other - differentiate at most their product - in order to relax price competition. This finding sharply contrasts with the acclaimed Principle of Minimum Differentiation of the original Hotelling model where firms, in the presence of linear transportation costs, choose to cluster in the product space. Examples of the tendency for competitors to reduce differences in distance or in the product characteristics space can be easily found in the real world. Conversely, examples of maximal differentiation can be identified in a truly locational perspective - e.g. the attitude for shopping centers and supermarkets to locate outside the urban center - but it is much more difficult to observe maximal differentiation in the characteristics space. The existence of a greater or a lower differentiation clearly depends on the interplay of a price competition effect and a demand effect: when the latter prevails, the firms' strategies are found to exhibit a strong tendency towards agglomeration in the middle; by contrast, when the demand effect is outweighed by the price competition effect, moving away is the optimal behaviour. Recently, this debate has been extended to cover situations with 'low' demand (Hinloopen and van Marrewijk 1999, Chirco, Lambertini and Zagonari, 2000) and to multi-dimensional models (Caplin and Nalebuff 1986, Neven and Thisse 1990, Tabuchi 1994).

One common property of traditional locational models is the assumption that consumers are uniformly distributed over the characteristics space; with a few exceptions, the situations in which the consumers' preferences are concentrated on a subsection of the available varieties have been neglected. In these cases one would expect that competitors produce fairly similar types of products, in order to better match the tastes of the relatively largest share of consumers (Beath and

Katsoulacos 1991). The problem of the optimal prices and locations has been explicitly solved by Tabuchi and Thisse (1995) with a triangular and symmetric distribution. They show that, given that distribution, any symmetric location around the middle cannot be an equilibrium. Indeed, two asymmetric equilibria arise, characterized by strong product differentiation between the firms, with one of them locating outside the support of the customer distribution.Their results, however, heavily depend on the non differentiability of the consumers density function, which generates a discontinuity of the reaction functions in correspondence of any symmetric location.

In this paper, we aim at extending Tabuchi and Thisse analysis in two directions. We offer a simple parametrization of the degree of consumers' concentration around the middle - which include the uniform and the triangular distribution as limit cases. This allows us to solve the price-location problem as a function of the degree of consumers concentration. Within this setup, we are able to show that a symmetric equilibrium exists, provided the density is differentiable at the center of its support. Moreover, we are able to give some theoretical support to the idea that a higher concentration of consumers around the center induces firms to reduce the optimal product differentiation. Finally, we find that the asymmetric equilibria identified by Tabuchi and Thisse may arise for a lower degree of consumers concentration than that implied by the triangular distribution and that these asymmetric equilibria may coexist with a symmetric one.

The paper is organized as follows. In section 2 we describe the basic model and discuss the simple parametrization of consumers' concentration adopted in the sequel. The explicit solution of the price-location problem is presented in section 3. Some comments and concluding remarks are provided in Section 4.

2 The model

Let us consider a market for a horizontally differentiated product, where the population of consumers is normalized to 1. Consumers, indexed with x, are distributed over the interval $[0,1]$, according to a density $f(x,w)$, where the parameter w that can be viewed as a concentration index of the consumers' tastes. More precisely, the density $f(x,w)$ is characterized as follows:

$$\begin{array}{lll} & f(x,1)=1, & \text{for } x\in[0,1] \\ & f(x,0)=2-2\left|2x-1\right|, & \text{for } x\in[0,1] \\ \text{for } 0<w<1 & f(x,w)=\dfrac{4}{1-w^2}x & \text{for } x<\dfrac{1-w}{2}, \\ & f(x,w)=\dfrac{2}{1+w} & \text{for } x\in\left[\dfrac{1-w}{2},\dfrac{1+w}{2}\right], \\ & f(x,w)=\dfrac{4}{1-w^2}(1-x) & \text{for } x<\dfrac{1+w}{2} \end{array}$$