assume symmetric conjectures λ on the reaction of the PMs of the same multiproduct firm, and symmetric conjectures μ on the reaction of the PMs of rival firms. Different values of the conjectural variations λ are equivalent to different internal organizational structures.

The paper is organized as follows. Section 1 describes the demand side of the model starting from a *compound* CES utility function. Under the assumption that each brand is produced by a mono-product firm, the market equilibrium is derived in section 2 through the use of different conjectural variations. The results are reinterpreted in section 3 in terms of optimal price-setting behavior of multi-product firms, where the organizational structure of the corporate firm is endogenous. In particular, it is shown that under market interlacing, independent PMs may be more profitable than a centralized GD. Some conclusions are gathered in section 4.

1 Preferences

Consider an economy with identical households. The economy produces a numéraire homogeneous good and $M \geq 1$ groups of differentiated goods. Each group consists of $n_i \geq 1$ $(i = 1, ..., M)$ varieties or brands (indexed by $j = 1, ..., n_i$, $\forall i$), so that the total number of varieties in the industry is $N = \sum_{i=1}^{M} n_i$.

Preferences are identical for all consumers. The representative household maximizes the utility function $U = U(x_0, V)$, where x_0 is the numéraire good and $U(\cdot)$ is homothetic in its arguments. Given this property, the utility maximization problem can be decomposed into two steps (Spence 1976). In particular, we assume that V has a *compound* CES functional form:

$$V(x_i) = \left[\sum_{i=1}^{M} x_i^{\alpha}\right]^{\frac{1}{\alpha}} \tag{1}$$

$$x_i(x_{ij}) = \left(\sum_{j=1}^{n_i} x_{ij}^{\beta}\right)^{\frac{1}{\beta}} \tag{2}$$

where x_{ij} is the quantity consumed of the *j-th* product of the *i-th* group and x_i represents the quantity index of the *i-th* group. Concavity of $V\left[x_i\left(x_{ij}\right)\right]$ requires that $0 < \alpha < 1$ and $0 < \beta < 1$[3]. This utility function implies a

[3] The love for variety could alternatively be modelled in a slightly different framework, by extending preferences over a continuous product space (Grossman and Helpman, 1989; Krugman, 1980).

constant elasticity of substitution between any couple of varieties of different groups of products (inter-group elasticity of substitution):

$$\sigma = \frac{1}{1-\alpha} > 1 \tag{3}$$

and a constant elasticity of substitution between any couple of varieties of the same group of products (intra-group elasticity of substitution):

$$\delta = \frac{1}{1-\beta} > 1 \tag{4}$$

Let's now denote with Y the consumer aggregate expenditure on the industry products and with p_{ij} the price of the ij-th variety. The consumer's problem becomes:

$$MAX_{x_{ij}} V(x_{11}, ..., x_{ij}, ..., x_{Mn_M}) = \left(\sum_{i=1}^{M} \left[\left(\sum_{j=1}^{n_i} x_{ij}^{\beta} \right)^{\frac{1}{\beta}} \right]^{\alpha} \right)^{\frac{1}{\alpha}} \tag{5}$$

$$s.t. \qquad Y = \left[\sum_{i=1}^{M} \left(\sum_{j=1}^{n_i} p_{ij} x_{ij} \right) \right]$$

First, the consumer maximizes $x_i\,(x_{ij})$ subject to the expenditure constraint on the products of the group i:

$$MAX_{x_{ij}} x_i = \left(\sum_{j=1}^{n_i} x_{ij}^{\beta} \right)^{\frac{1}{\beta}} \tag{6}$$

$$s.t. \qquad Y_i = \sum_{j=1}^{n_i} p_{ij} x_{ij}$$

where $\sum\limits_{i=1}^{M} Y_i = Y$ and Y_i represents the total expenditure on the i-th group. In the second step, the household maximizes the utility V as a function of x_i, subject to the budget constraint on the overall of the M groups:

$$MAX_{x_i} V = \left(\sum_{i=1}^{M} x_i^{\alpha} \right)^{\frac{1}{\alpha}} \tag{7}$$

$$s.t. \qquad Y = \sum_{i=1}^{M} x_i q_i$$

where q_i, the price-index corresponding to the *i-th* group, is given by:

$$q_i = \left(\sum_{j=1}^{n_i} p_{ij}^{1-\delta} \right)^{\frac{1}{1-\delta}} \tag{8}$$

and q, the industry price-index, is given by:

$$q = \left(\sum_{i=1}^{M} q_i^{1-\sigma} \right)^{\frac{1}{1-\sigma}} \tag{9}$$

The solution to the first step gives us $x_{ij}(Y_i, p_{ij}) = \frac{Y_i}{p_{ij}} \left(\frac{p_{ij}}{q_i} \right)^{1-\delta}$, while the two-stage budgeting procedure requires that $Y = Y_i \left(\frac{q_i}{q} \right)^{\sigma-1}$.

Therefore, the demand schedule for the *j-th* brand in the *i-th* group ($\forall j \in [1, n_i]$; $\forall i \in [1, M]$) is:

$$x_{ij}\left(p_{ij}\right) = \frac{Y}{p_{ij}} \left(\frac{p_{ij}}{q_i} \right)^{1-\delta} \left(\frac{q_i}{q} \right)^{1-\sigma} \tag{10}$$

Equation (10) will be used in the analysis of firms' price-setting behavior. If we are interested in quantity competition, we may consider the corresponding inverse demand function:

$$p_{ij}\left(x_{ij}\right) = \frac{Y}{x_{ij}} \left(\frac{x_{ij}}{Q_i} \right)^{\beta} \left(\frac{Q_i}{Q} \right)^{\alpha} \tag{11}$$

where Q_i is the quantity-index of group i, given by:

$$Q_i = \left(\sum_{j=1}^{n_i} x_{ij}^{\beta} \right)^{\frac{1}{\beta}} \tag{12}$$

and Q is the industry quantity-index, given by:

$$Q = \left[\sum_{i=1}^{M} Q_i^{\alpha} \right]^{\frac{1}{\alpha}} \tag{13}$$

Notice the immediate interpretation of α and β in terms of the structure of preferences. The parameters are indicators of the degree of substitutability between any couple of varieties: the lower α, the lower the interdependence among varieties produced in different groups; the higher β, the higher the interdependence among brands produced within the same group. At the intra-group level, as $\delta \to 1$ the degree of product differentiation is maximum. As $\delta \longrightarrow \infty$, there is no intra-group differentiation, the degree of substitutability becomes infinite and varieties of the same group are homogeneous. In the inter-group perspective, as $\alpha \rightarrow 0$ the degree of substitution reaches the minimum level (i.e. $\sigma \rightarrow 1$) and varieties of different groups become highly differentiated. As $\alpha \rightarrow 1$ there is no differentiation between varieties of different groups, the degree of substitutability becomes infinite (i.e. $\sigma \longrightarrow \infty$) and any brand is perfectly substitutable with any others of the remaining $M-1$ groups.

Moreover, the difference between σ and δ plays a fundamental role: when the inter-group elasticity of substitution is greater than the intra-group elasticity of substitution (i.e. $\sigma > \delta$), brands of different groups are closer substitutes rather than varieties of the same group; on the contrary, when $\sigma < \delta$ each variety is more substitutable with a brand of the same group. This aspect is also captured by demand elasticities. In this model three price elasticities are defined: with respect to the own price, with respect to the price of brands produced in the same group and with respect to the price of goods produced in different groups. Let us denote with $\eta_{x_{ik},p_{ij}}$ the cross price elasticity of the demand function of the *ik-th* variety with respect to the price of a brand of the same group, p_{ij}; and with $\eta_{x_{hk},p_{ij}}$ the cross price elasticity of the demand function of the *hk-th* variety with respect to the price of any other brand of a different group, p_{ij}. The demand elasticity of the *ij-th* variety with respect to the own price, p_{ij}, is:

$$\eta_{x_{ij},p_{ij}} = -\frac{\partial x_{ij}}{\partial p_{ij}}\frac{p_{ij}}{x_{ij}} = \delta + (\sigma - \delta)\left(\frac{p_{ij}}{q_i}\right)^{1-\delta} + (1-\sigma)\left(\frac{p_{ij}}{q_i}\right)^{1-\delta}\left(\frac{q_i}{q}\right)^{1-\sigma} \tag{14}$$

The cross price elasticity of the demand of the *ik-th* variety with respect to the price of a brand produced within the same group, p_{ij}, can be written as:

$$\eta_{x_{ik},p_{ij}} = \frac{\partial x_{ik}}{\partial p_{ij}}\frac{p_{ij}}{x_{ik}} = -(\sigma - \delta)\left(\frac{p_{ij}}{q_i}\right)^{1-\delta} - (1-\sigma)\left(\frac{p_{ij}}{q_i}\right)^{1-\delta}\left(\frac{q_i}{q}\right)^{1-\sigma} \tag{15}$$

The cross price elasticity of the demand of the *hk-th* variety with respect to the price of any other brand of a different group, p_{ij}, is:

$$\eta_{x_{hk},p_{ij}} = \frac{\partial x_{hk}}{\partial p_{ij}}\frac{p_{ij}}{x_{hk}} = -(1-\sigma)\left(\frac{p_{ij}}{q_i}\right)^{1-\delta}\left(\frac{q_i}{q}\right)^{1-\sigma} \tag{16}$$

Notice that these elasticities correspond to the elasticities perceived by the price-maker for the relevant variety, when each of them believes that his decisions do not affect the rivals' price decisions[4] but takes into account the effect of his price on the relevant price indices. If $\eta_{x_{ik},p_{ij}} < \eta_{x_{hk},p_{ij}}$ (i.e. $\sigma > \delta$), then x_{ij} and x_{ik} are distant substitutes, while x_{ij} and x_{hk} are closer substitutes.

[4] That is in case each firm has a standard Bertrand conjectures. We shall generalize this assumption making use of non-zero conjectural variations in the next section.

If the opposite holds (for $\sigma < \delta$, i.e. $\eta_{x_{ik},p_{ij}} > \eta_{x_{hk},p_{ij}}$), x_{ij} and x_{ik} are close substitutes, while x_{ij} and x_{hk} are distant substitutes [5] .

1.1 An example

In the above discussion, the varieties produced in the market are collected in groups, which may consist of either close substitutes or distant substitutes. In order to clarify this point an example may be of help.

Consider the *Carbonated Beverages* market. In this market it is possible to group the products according to two criteria. The first is based on intrinsic characteristics of the products themselves: e.g. *Fruits* drinks, *Cola* drinks and *Fizzy* drinks. In this case each group consists of close substitutes. The *Fruits* drinks are *Fanta*, *Oransoda*, and *Lemonsoda* etc.; in the second group we find *Coke*, *Pepsi*, and *Virgin*; while the last group is made by drinks such as *Sprite*, *Schweppes*, and *Seven-Up*.

According to a second criterion, however, it is possible to collect products on the basis of the different trade-marks under which the varieties are sold. In this case we have the group of the *Coca-Cola Company* which produces distant substitutes, such as *Coke*, *Sprite*, *Fanta*; the same occurs for the *PepsiCo International Inc.* which sells *Pepsi-Cola*, *Mountain-Dew* and *Slice-Soda*. On the other side,under their trade-marks both *Cadbury-Schweppes plc.* and *Gruppo-Campari* produce close substitutes. For example, in the product line of the former we can find *Seven-Up*, *dnL*, *Schweppes*, while the latter produces *Lemonsoda*, *Oransoda*, *Pelmosoda* and *Tonicsoda*.

In section 3, the product line of a multiproduct firm will coincide with a *'group'* in the above definition. This allows to cover both the situations which Brander and Eaton (1984) define as market segmentation - the multiproduct firm produces close substitutes (e.g. the *Cadbury-Schweppes plc.* and *Gruppo-Campari* cases) - and those of market interlacing - each company produces distant substitutes (the *Coke-Cola* and *Pepsi-Cola* examples).

[5] The substitutability relationship may be expressed using direct or inverse demand function, and using either elasticities or derivatives. Moreover, the definitions do not necessary coincide. The concept of substitutability can be definied in terms of "q-substitutes" (with reference to the inverse demand function), or in terms of "p-substitutes" (with reference to the direct demand function) (Hicks (1956)). Since in the next sections we shall assume that prices are the firms' strategic variable, the p-substitutes approach is more convenient.