or, equivalently, when

$$n_h\gamma_h^{1-\sigma}\sum_{j=1}^{h} n_j\gamma_j^{1-\sigma} + n_h\gamma_h^{1-\sigma} n_{h+1}\gamma_{h+1}^{1-\sigma} - n_{h+1}\gamma_{h+1}^{1-\sigma}\sum_{j=1}^{h} n_j\gamma_j^{1-\sigma} > 0 \tag{40}$$

Expression (40) is true when

$$\frac{\sum_{j=1}^{h} n_j\gamma_j^{1-\sigma}}{\sum_{j=1}^{h-1} n_j\gamma_j^{1-\sigma}} > \frac{n_{h+1}\gamma_{h+1}^{1-\sigma}}{n_h\gamma_h^{1-\sigma}}$$

We substitute $n_{h+1}$ from (21) and we obtain

$$l \equiv \frac{\sum_{j=1}^{h} n_j\gamma_j^{1-\sigma}}{\sum_{j=1}^{h-1} n_j\gamma_j^{1-\sigma}} > \frac{L_R\gamma_{h+1}^{1-\sigma}}{a\gamma_h^{1-\sigma}} = \frac{L_R}{a}\left(\frac{\gamma_h}{\gamma_{h+1}}\right)^{\sigma-1}$$

where the left term in the inequality, $l$, is always larger than 1. Therefore, given that $\gamma_{h+1} < \gamma_h$, we may at least state that $b_h > b_{h+1}$ is true, when $\frac{L_R}{a}\left(\frac{\gamma_h}{\gamma_{h+1}}\right)^{\sigma-1} < l$. That is when

$$1 < \left(\frac{\gamma_h}{\gamma_{h+1}}\right)^{\sigma-1} < \frac{a}{L_R} l \tag{41}$$

Expression (41) says that when the process innovation produces a reduction in $\gamma$ which is not relatively high, then $b_i$ decreases.

## Appendix B

Following Grossman and Helpman (1991, p. 63) we define the index of the manufactured output

$$D \equiv \left(\sum_{m=1}^{i} n_m x_m^{\alpha}\right)^{\frac{1}{\alpha}}$$

where $\alpha = \frac{\sigma-1}{\sigma}$, while the ideal price index of final goods is $p_D$.

The gross domestic product (GDP), $G$, is defined as the sum of the value added in manufacturing and in the R&D sector

$$G \equiv p_D D + v_i \dot{n}_i$$

We know from Grossman and Helpman (1991, p. 63) that the growth of the real GDP is equal to a weigthed average of the growth rates of the manufactured good index, $g_D$, and of the research output, $g_i$, with weights given by sector's value shares. In particular, the manufactured goods share is given by $\theta_D \equiv p_D D / (p_D D + v_i \dot{n}_i)$. Thus the growth rate of the real GDP is

$$g_G = \theta_D g_D + (1 - \theta_D) g_i$$

We need to compute $g_D$ for a given value of $b_i$.

Using (29), (26) and (17), we rewrite $D$ as follows

$$\begin{aligned}
D^{\alpha} &\equiv \sum_{m=1}^{i} n_m \left( \frac{\alpha b_m}{n_m \gamma_m} \right)^{\alpha} w^{-\alpha} = \sum_{m=1}^{i} n_m \left( \frac{\alpha \frac{n_m \gamma_m^{1-\sigma}}{\sum_{j=1}^{i} n_j \gamma_j^{1-\sigma}}}{n_m \gamma_m} \right)^{\alpha} w^{-\alpha} = \\
&= \sum_{m=1}^{i} n_m \left( \alpha \frac{\gamma_m^{-\sigma}}{\sum_{j=1}^{i} n_j \gamma_j^{1-\sigma}} \right)^{\frac{\sigma-1}{\sigma}} w^{-\alpha} = \frac{\alpha^{\frac{\sigma-1}{\sigma}} \sum_{m=1}^{i} n_m \gamma_m^{1-\sigma}}{\left( \sum_{j=1}^{i} n_j \gamma_j^{1-\sigma} \right)^{\frac{\sigma-1}{\sigma}}} w^{-\alpha} = \\
&= \alpha^{\alpha} \left( \sum_{m=1}^{i} n_m \gamma_m^{1-\sigma} \right)^{\frac{1}{\sigma}} w^{-\alpha} = \alpha^{\alpha} \left( \sum_{m=1}^{i} n_m \gamma_m^{1-\sigma} \right)^{1-\alpha} w^{-\alpha}
\end{aligned}$$

Therefore

$$D^{\frac{\alpha}{1-\alpha}} \equiv \alpha^{\frac{\alpha}{1-\alpha}} \left( \sum_{m=1}^{i} n_m \gamma_m^{1-\sigma} \right) w^{-\frac{\alpha}{1-\alpha}}$$

and totally differentiating the previous expression, we obtain

$$\begin{aligned}
\frac{\alpha}{1-\alpha} D^{\frac{\alpha}{1-\alpha}-1} \dot{D} &= -\frac{\alpha}{1-\alpha} \alpha^{\frac{\alpha}{1-\alpha}} w^{-\frac{\alpha}{1-\alpha}-1} \dot{w} \left( \sum_{m=1}^{i} n_m \gamma_m^{1-\sigma} \right) + \alpha^{\frac{\alpha}{1-\alpha}} w^{-\frac{\alpha}{1-\alpha}} \gamma_i^{1-\sigma} \dot{n}_i \\
g_D &= -\hat{w} + \frac{1-\alpha}{\alpha} \frac{n_i \gamma_i^{1-\sigma}}{\left( \sum_{m=1}^{i} n_m \gamma_m^{1-\sigma} \right)} g_i \\
g_D &= -\hat{w} + \frac{(1-\alpha)}{\alpha} b_i g_i
\end{aligned}$$

We know from (17) that

$$\hat{w} = -\hat{V}_i$$

Since for any given value of $b_i$ we know that $\hat{V}_i = 0$, we derive that

$$g_D = \frac{(1-\alpha)}{\alpha} b_i g_i$$

Moreover, given our normalization for manufacturing expenditure, we know that $E = p_D D = 1$ and

$$\theta_D = \frac{1}{1 + \frac{1}{V_i} g_i} \tag{42}$$

Expression (42) tells us that the manufactured goods share, $\theta_D$, is constant if $V_i$ is constant. We know that $V_i$ is constant only if $b_i$ does not change. Consequently, the real GDP growths at the following rate

$$g_G = \left[\theta_D \frac{(1-\alpha)}{\alpha} b_i + (1-\theta_D)\right] g_i$$

which is constant when $b_i$ is constant, given that we know from (37) that also $g_i$ is constant.