deficit? Is this acceptable?) and how the tax cut affect the economic growth, rather than which distribution of income.

2 – A Progressive Income Tax Schedule: Theoretical Results

The large majority of European and OECD countries have a progressive income tax. Given the respect, or not, of horizontal equity (HE) prior principle (a normative goal) "progression arises from principles of vertical equity" (Lambert, 2001, p. 175): vertical equity states taxpayers in unequal circumstances bear appropriately unequal taxes (a matter of judgment, that is an ethical norm referring to the treatments of unequals)[2].

The literature suggests some rationales, for instance the Equal sacrifice principle (by using an increasing concave utility of income function equal for all individuals, this calls for a equal utility loss for all)[3] as well as the Benefit principle (by relating the tax liability to the expenditure side of budget)[4]; the automatic stabilizing effect on the macro-economy; the envy reducer capacity.

After the seminal Atkinson (1970) and Shorrocks' (1983) papers another way to justify progression was on hand to the researchers: I shall give an idea about it referring to the papers where it is opportune. Before to do that I provide some definitions about what it defines as a progressive income tax as well as some other useful instruments.

2.1 Some Definitions

An income tax schedule embodies a succession of upward and fixed marginal tax rates (the *rate structure*) on bands of taxable income with different specified threshold values.

If $t(\mathrm{x})$ is the tax liability[5] of the pre tax money income x and is a differentiable function, an increasing with income average tax rate $t(\mathrm{x})/\mathrm{x}$ is the condition for strict progression

$$d\,[t(\mathrm{x})/\mathrm{x}]\,/\,d\mathrm{x} > 0 \quad \forall \mathrm{x} \quad \text{iff} \quad t'(\mathrm{x}) > t(\mathrm{x})/\mathrm{x} \quad \forall \mathrm{x} > 0$$

In this case the tax burden is function only of money incomes while the typical real income tax code usually is also function of other features: we shall see how to take account of these non-income characteristics.

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[2] - In a second best-world there is a potential trade-off between HE and VE objectives (see Musgrave 1990).
[3] - See Samuelson (1947).
[4] - See Head (1964), Snow and Warren (1983).
[5] - I assume $0 \leq t'(\mathrm{x}) < 1$ (incentive preservation principle), then $0 \leq t(\mathrm{x}) < \mathrm{x}$ (where $t'(\mathrm{x})$ is the first derivative).

Let F(x) be the pre tax distribution of income and f(x) the associated density function, then (assuming no convergence problem at the top end of the distribution)

$$\text{Total pre tax income} = X = N \int_0^z x\, f(x)\, dx$$

$$\text{Total income tax revenue} = T = N \int_0^z t(x)\, f(x)\, dx$$

$$\text{Total tax ratio} = T/X = g = N \int_0^z t(x)\, f(x)\, dx \,/\, N\mu = \int_0^z t(x)\, f(x)\, dx \,/\, \mu$$

where $\mu = \int_0^z x\, f(x)\, dx$ and z could be described as 'any income level in excess of the highest one actually occurred' (Lambert, 2001, p. 20)

According to these integrals it can derive definitions for the Lorenz curve for pre tax income L_X and the Concentration curves for post–tax income and tax liabilities, L_{X-T} and L_T [6].

$$p = F(y) \rightarrow L_X(p) = [1/\mu] \int_0^y x\, f(x)\, dx$$

$$p = F(y) \rightarrow L_{X-T}(p) = [1/\mu(1-g)] \int_0^y [x - t(x)]\, f(x)\, dx$$

$$p = F(y) \rightarrow L_T(p) = [1/\mu g] \int_0^y t(x)\, f(x)\, dx$$

By using these definitions it derives the relationship between L_X, L_{X-T} and L_T

$$L_X \equiv g\, L_{X-T} + (1\text{-}g)\, L_T$$

The L_X curve is a weighted average of tax and post tax income Concentration curves: as a result it can be shown that if and only if the tax burdens are distributed more unequally than before-tax incomes, the post tax incomes shares are more equal than the latter. Where L_X and L_T curves are superimposed also L_{X-T} has to be it.

Furthermore Jakobsson (1976) and Fellman (1976) point out that this relationship is strictly linked with the progression of the tax schedule:

$$d\,[t(x)/x] \,/\, dx \geq 0 \quad \forall x \quad \text{iff} \quad L_{X-T} \geq L_X \geq L_T \quad \forall\, F(x)$$

Thus, with a tax code designed for any sub-population where the only differences among people are the incomes, a strictly progressive income tax is within group inequality reducing according to the dominance of post tax income concentration curve over the pre tax income Lorenz curve. According to $t(x)$ function only of money incomes I approach the normative issue.

[6] - A Lorenz Curve measures income shares plotting them against cumulative proportion of income units in the pre tax distribution. The units are arranged in ascending order of their income. The assumptions on the form of $t(x)$ function (5th footnote) allow to consider the L_{X-T} concentration curve as a Lorenz curve (no reranking effect).

2.2 Progression, Progressivity and Welfare

According to Atkinson (1970) a dominance result by comparisons of Lorenz curves may fail: not any pair of income distributions can be clearly ranked (the Lorenz ordering is a partial ordering). By the Shorrocks (1983) theorem, sometimes it may go beyond the Lorenz curve comparison failing: by using generalized Lorenz curve concept it could be possible to rank unambiguously more (not any) distributions of income.

With these theorems it is possible to describe why progression should be a "good thing", that is to verify if the welfare related with a progressive income tax is higher than the welfare associated with another way to raise the *same* revenue, in this case an equal yield flat tax applied to the *same* distribution of before tax incomes.

According to the fact that generalized Lorenz curves (by the Jakobsson-Fellman theorem) for post tax incomes, when progressive income tax and equal yield flat tax apply, are respectively[7]

$$p = F(y) \rightarrow GL_{X-T}(p) = \mu(1-g)\int_0^y [x - t(x)]\, f(x)\, dx / \mu(1-g) = \mu(1-g)\, L_{X-T}(p)$$

$$\text{and } p = F(y) \rightarrow GL_{X(1-G)}(p) = \mu(1-g)\int_0^y x\, f(x)\, dx / \mu = \mu(1-g)\, L_X(p)$$

it can be inferred that the distribution of post tax incomes associated with a progressive income tax rank dominates the distribution of post tax incomes related with the proportional tax. By Shorrocks theorem the dominance between *GL* curves implies (and it's implied by) a higher level of Social Welfare reached by the dominant position:

$$W_{X-T} \geq W_{X(1-G)} \qquad \text{for all increasing and strictly concave } U(\bullet)$$[8]

This shows that choosing progressive income taxation rather than a proportional tax it obtains the *smaller* decrease of Social Welfare.

Hence the literature has developed measures of structural (or *local*) progression and *effective* progression, the former linked with the degree of progression along the income scale, the latter with the taxation effects on the after-tax income distribution.

Both give information about the consequences of different tax income schedules on their disproportionality degree and resultant redistributive effects[9] and what is important for my

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[7] - By (x–*t*x) = post-tax income, when equal-yield flat tax applies: the net incomes means in both cases are equal.

[8] - Where $W_F = \int_0^z U(x)\, f(x)\, dx$

[9] - See Jakobsson (1976) and Kakwani (1977a). Note that, advocating the fundamental Atkinson theorem (1970), Formby and Smith (1986, p. 562) comment, "If Lorenz curves intersect, a social welfare function can always be found which ranks income distribution differently than does the Gini coefficient or other summary measures of

purposes is the possibility, by using measures of *effective* progression, to obtain significant conclusions about the level of Social Welfare that different tax schedules incorporate.

Usually when the analysts wish to evaluate it they first look at two indexes, the Reynolds-Smolensky (Π^{RS}) and the Kakwani (Π^{K}) indexes: they are scalar index numbers, able to resume a tax code and income distribution pair. They are defined in terms of separation of the relevant Lorenz curves (the higher the separation between the curves, the higher the index),

$$\Pi^{K} = 2\int_0^1 [L_X(p) - L_T(p)]\,dp \quad ; \quad \Pi^{RS} = 2\int_0^1 [L_{X-T}(p) - L_X(p)]\,dp$$

or, in a short way, $\Pi^{K} = C_T - G_X$; $\Pi^{RS} = G_X - C_{X-T}$

where G_X is the Gini coefficient for before-tax income, C_T and C_{X-T} (here, $C_{X-T} = G_{X-T}$) the concentration coefficients respectively for tax liabilities and post tax income. The higher is Π^{RS}, the 'more' dominant is the distribution of post tax incomes over the distribution defined by L_X, where as before the latter has to be interpreted as the distribution of after-tax incomes resulting from an *equal yield* flat tax.

Clearly it has to be a link between the Π^{RS} and Π^{K} indexes strictly connected with the identity that represent the L_X curve as a weighted average between L_{X-T} and L_T.

By Kakwani (1977b) it can be shown that (if no reranking through the taxation process is present)

$$\Pi^{RS} = [g / (1 - g)]\, \Pi^{K}$$

What result from the Social Welfare point of view? Do we can conclude that a 'more' progressive tax is a better tax? For instance, given the assumptions made and choosing the social evaluation function (where μ and G are the mean and the Gini coefficient (Sen 1973)),

$$V(\mu, G) = \mu\,(1-G)$$

it can be shown that[10]

$$W_{X-T} - W_{X(1-g)} = \mu\,(1-g)\,[(1 - G_{X-T}) - (1 - G_X)]$$

$$W_{X-T} - W_{X(1-g)} = \mu\,(1-g)\,\Pi^{RS}$$

$$W_{X-T} - W_{X(1-g)} = \mu\, g\, \Pi^{K}$$

In both cases the higher is the relevant index the greater is the Welfare Premium from progression.

inequality.". As a consequence, if Lorenz curves do not intersect, any inequality index that fulfil the Pigou-Dalton transfer principle and Symmetry will be robust.

[10] - By the equality $C_{X-T} = G_{X-T}$.

2.3 Income Tax Reforms

The results described in the last paragraph allow to analyse any personal tax income reform: by means of a sample of the distribution of before-tax incomes it can make an estimate to predict the consequences for distributional effects, the disproportionality of the new tax burden and, where feasible, the question of normative significance. Assuming a fixed distribution of pre tax incomes F(x), there are three alternative cases according to a change in a tax schedule t(x):

a yield neutral/increasing/decreasing tax reform according as g (total tax ratio) is equal/ increased/decreased after the reform

Also the measures of structural progression[11] Π^{RS} and Π^{K} modify in accordance with this change.

The consequences for *g* and indexes may be linked and operate together, turning out different overall results of the reform[12]. Consistent with these, the literature is able to provide to practitioners normative assessments about some general cases:

- a progression neutral tax cut/hike for all;
- a not[13] progression neutral tax cut/hike for all;
- a single-crossing tax reform and a double-crossing tax reform.

About a progressivity-neutral reform, consider a 'linear' income tax cut/hike preserving some local progressivity measure such as RP and LP[14]. From the old schedule $t^1(x)$ it has to be, for the RP case

$$t^2(x) = t^1(x) - a\,[x - t^1(x)] \quad ; \quad x - t^2(x) = [x - t^1(x)] + a\,[x - t^1(x)]$$

and for the LP case

$$t^2(x) = t^1(x) - b\,t^1(x) \quad ; \quad x - t^2(x) = [x - t^1(x)] + b\,t^1(x)$$

where $t^2(x)$ is the new schedule and *a* and *b* are positive/negative constant parameters according to a tax cut/hike.

Following Pfähler (1984) if we have to choose between these reforms, the best way to share between income units the required changes in the income tax yield is to follow a RP neutral policy if we apply a tax cut and a LP neutral policy for the tax hike. In this way post tax income inequality is less and social welfare is higher, according to the Lorenz (or, more

[11] - See Jakobsson (1976) and Kakwani (1977a).

[12] - Realistically we can exclude the case of equality of g (pre and post reform) with the constancy of two indexes, although this is theoretically possible.

[13] - From the empirical evidence, we shall see that this seems to be the case of the Italian tax reform proposal.

[14] - Residual progression = RP = $e_{x - t(x),\, x}$ = $(d\,(x - t(x)) / dx)\,(x / x - t(x))$; Liability progression = LP = $e_{t(x),\, x}$ =

precisely, Concentration) dominance criterion and Atkinson theorem (it can be shown those two curves for post tax incomes, or taxes - L_{X-T} or L_T - do not intersect and have the same mean for both).

Looking at a single-crossing reform, as before the revenue could be neutral, increasing or decreasing. About the distributional effects, there are two cases: the old and new tax schedule cross once, the new one crossing the old one from below or from above. The former leads to redistribution from rich to poor, for the latter the opposite is the case.

For instance, according to Dardanoni-Lambert (1988), it is possible to give some normative evaluation when the tax reform is inequality reducing:

[**1**] for any before-tax distribution : $g^2 \leq g^1$ (by $t^1(x) \rightarrow t^2(x)$) and the new schedule crosses the old one from below. The mean of post tax incomes is higher or equal, the redistribution operates to reduce inequality (accepting the Dalton's Transfer Principle), hence the *GL* curve after the tax reform is higher relatively to the *GL* curve for the old schedule and social welfare level follows by Shorrock's theorem: it is higher for tax reform, for all increasing and strictly concave U(•);

[**2**] for any before-tax distribution : $g^2 > g^1$ (by $t^1(x) \rightarrow t^2(x)$) and the new schedule crosses the old one from below.

The conditions are more stringent than [**1**] because $GL_{X-T}(p)$ after the tax reform crosses the old $GL_{X-T}(p)$ from above (inequality reducing but with a lower mean, $\mu(1-g^2) \leq \mu(1-g^1)$, then with an inferior efficiency): to approve the tax reform normatively we need to decrease 'enough' the variance in the 'new' distribution of post tax incomes.

With reference to reform where a *non*-progression neutral tax cut/hike for all applies, I postpone the analysis to the last section of this essay whereas, concerning a double-crossing reform (of new and old tax schedule) justifiable by welfare approval, we refer to Lambert (2001, pp. 231-235).

2.4 The Typical Income Tax: Social Heterogeneity

Until now we are assessing by using some uncomplicated basis:

a) non-income factors are completely neglected by income tax schedules, they apply only on money incomes;

(d t(x) / dx) (x / t(x)) ; Average rate progression = ARP = d(t(x)/x) / dx.

b) as a result every individual utility, heart of the Welfare 'measure' (the average utility in society), is function only of money incomes;

c) $0 \leq t\,(\mathrm{x}) < \mathrm{x},\ 0 \leq t'(\mathrm{x}) < 1$;

d) the allocation of pre tax incomes is given: a partial equilibrium analysis where incentive effects are excluded by construction (ceteris paribus condition).

As a result of the assumption *a)*, we employed a personal tax schedule $t(\mathrm{x})$ where x is pre–tax money income: this could be considered a correct procedure only if all households are equal in size (and, possibly, composition)[15] that is, if there is social homogeneity. Clearly this looks an improbable case and in the real world tax systems should take into account this matter.

The usual way used by practitioners is to apply an equivalence scale to deflate *appropriately* household money incomes, to have a common base of measure and rise above the heterogeneity problem. The equivalence scale uses a coefficient (z_n) for each type of family size and relative composition and it obtains a distribution of *equivalent* pre tax income (and a distribution of *equivalent* post tax income) dividing every households money income by these coefficients. They *should* reflect the differing needs of the relevant households: this is a matter of value judgment and it is not unusual to find differing opinions about the fairness of the coefficients of applied equivalence scales (moreover, how do they take full account of all non-income significant factors?).

After this type of conversion, only now it can apply all the theoretical and social welfare results described above: from Ebert (1997, 1999) it can be shown that the best way, especially for the distributional analysis, is first to construct a *standard* individual, an artificial population of income units, substituting an household of size n "by a set of z_n equivalent adults" .

By using this *fictional* individual concept the literature showed (Lambert, 2001, par. 10.1) which has to be the right income tax procedure when the population of households is socially heterogeneous. The goal is to avoid non-horizontally inequitable households' income tax. To ensure this, families with equal pre tax utility level must confirm this equality also after the taxation.

Unfortunately many problems may take place and is very likely that, by a typical income tax, unequal treatment of equals rise up: this occurs when the tax system does not use a $t(\mathrm{x})$ function with $0 \leq t'(\mathrm{x}) < 1$ (homogenous case), or when $t_{\mathrm{n}}\,(\mathrm{x}) = z_n\, \tau_n\,(\,\mathrm{x}\,/\,z_n\,)$ and τ_n is not the same schedule for all the families (heterogeneous case). In these cases:

[15] - It seems obvious that a family of size $n \neq 1$ where only an individual gains should be treated in a different way relatively to a single person household: starting from equal pre-tax incomes situation, we have different level of pre tax purchasing power in these families and it is the living standard that is linked with the well-being.

- in all formulas defined on this section, it has to use equivalent income terms (the living standard);
- all the formulas derive results about households, not individuals;
- L_{X-T} is no more a Lorenz curve but a concentration curve; to define the position of the post tax Lorenz curve it has to take into account a correction factor reflecting the extent of reranking (R): when R is at work a difference between the pre tax and post tax rankings of (equivalent) income units is present and the post tax Lorenz curve is always dominated by the L_{X-T}. The original measures of effective progression change relationship, from[16]

  $$\Pi^{RS} = g / (1 - g)\, \Pi^{K} \qquad \text{to} \qquad \Pi^{RS} = g / (1 - g)\, \Pi^{K} - R$$

  It can refine the decomposition of the Reynolds-Smolensky index, capturing as well as the Kakwani index an effect (negative) of classical horizontal inequity[17] not present if equals had been treated equally;
- it still holds the last assumption of this section.

Now, it is time to verify if these instruments are useful to assess, where feasible, the Italian income tax reform using the empirical evidence.

3 - Empirical Evidence: First Results

The first stage of the Italian income tax reform started in 2003 (then, with the income tax document of 2004). The Executive method is proceeding by various steps: in accordance with the parliamentary majority objectives, at the beginning the tax cut has been concentrated on the bottom part of income parade (Bosi and Baldini 2004). Clearly only at the end of the transition period, the characteristics and the redistributive effect of the tax reform will be definitive.

All papers focus on the government proposal, but during period between 2000 up to July 2002 distinct information about the design of tax reform became available for the analysts: for instance, the first paper was on hand in 2000 and redistributive results went out according to the electoral plan of the upcoming government.

During time doubts are decreased by an improved definition of the proposal and, finally, by the appearance of 'official' case studies simulating the tax reform effects on individual tax liability, average rates and post tax income.

It can divide papers into three categories, according to different available information

[16] - See Kakwani (1984, 1986).
[17] - See Aronson et al. (1994).