Appendix A

Matrix Green function

The matrix Green function $\mathbf{G}(z, \xi)$ introduced in (5.41) is given by

$$\mathbf{G}(z, \xi) = \begin{pmatrix} G_{11}(z, \xi) & 0 \\ 0 & G_{22}(z, \xi) \end{pmatrix}$$

where

$$G_{11}(z, \xi) = \begin{cases} g_{11}(z, \xi) & \text{if } z \leq \xi \\ g_{11}(z, \xi) + \dfrac{a \sinh \alpha(z - \xi) - \alpha \sinh a(z - \xi)}{a\alpha \tilde{D}a R_m(\alpha^2 - a^2)} & \text{if } z > \xi, \end{cases}$$

$$G_{22}(z, \xi) = \begin{cases} g_{22}(z, \xi) & \text{if } z \leq \xi \\ g_{22}(z, \xi) + \dfrac{a(z - \xi) \cosh a(z - \xi) - \sinh a(z - \xi)}{2\tilde{D}a P_m a^3} & \text{if } z > \xi, \end{cases}$$

with $\alpha = \sqrt{\dfrac{1 + a^2 \tilde{D}a}{\tilde{D}a}}$,

$$g_{11}(z, \xi) = \frac{\alpha \sinh a(1 - \xi) - a \sinh \alpha(1 - \xi)}{a\alpha \tilde{D}a R_m(\alpha^2 - a^2)}$$

$$\cdot \left\{ \frac{a\alpha[\cosh a(1 - z) - \cosh 2\alpha \cosh a(z + 1)] + \alpha^2 \sinh 2\alpha \sinh a(z + 1)}{2a\alpha(1 - \cosh 2\alpha \cosh 2a) + (a^2 + \alpha^2) \sinh 2\alpha \sinh 2a} \right.$$

$$\left. + \frac{a\alpha[\cosh \alpha(1 - z) - \cosh 2a \cosh \alpha(z + 1)] + a^2 \sinh 2a \sinh \alpha(z + 1)}{2a\alpha(1 - \cosh 2\alpha \cosh 2a) + (a^2 + \alpha^2) \sinh 2\alpha \sinh 2a} \right\}$$

$$+ \frac{\cosh a(1 - \xi) - \cosh \alpha(1 - \xi)}{\tilde{D}a R_m(\alpha^2 - a^2)}$$

$$\cdot \left\{ \frac{a \sinh 2\alpha \cosh a(z + 1) - \alpha[\sinh a(1 - z) + \cosh 2\alpha \sinh a(z + 1)]}{2a\alpha(1 - \cosh 2\alpha \cosh 2a) + (a^2 + \alpha^2) \sinh 2\alpha \sinh 2a} \right.$$

$$\left. + \frac{\alpha \sinh 2a \cosh \alpha(z + 1) - a[\sinh \alpha(1 - z) + \cosh 2a \sinh \alpha(z + 1)]}{2a\alpha(1 - \cosh 2\alpha \cosh 2a) + (a^2 + \alpha^2) \sinh 2\alpha \sinh 2a} \right\}$$

and

$$
\begin{aligned}
g_{22}(z,\xi) = {} & \frac{\sinh a(1-\xi) - a(1-\xi)\cosh a(1-\xi)}{2\tilde{D}aP_m a^3(4a^2 - \sinh^2 2a)}\big\{a^2\cosh a(1-z) \\
& - [(a^2+1)\sinh 2a + 2a\cosh 2a]\sinh a(z+1) \\
& + a(\sinh 2a + a\cosh 2a)\cosh a(z+1)\big\} \\
& + \frac{(1-\xi)\sinh a(1-\xi)}{2\tilde{D}aP_m a(4a^2 - \sinh^2 2a)}\big\{a\sinh a(1-z) \\
& - (\sinh 2a + a\cosh 2a)\sinh a(z+1) + a\sinh 2a\cosh a(z+1)\big\} \\
& + z\frac{\sinh a(1-\xi) - a(1-\xi)\cosh a(1-\xi)}{2\tilde{D}aP_m a^2(4a^2 - \sinh^2 2a)}\big\{a\cosh a(1-z) \\
& + \sinh a(1-z) \\
& + (\cosh 2a - a\sinh 2a)\sinh a(z+1) + a\cosh 2a\cosh a(z+1)\big\} \\
& + z\frac{(1-\xi)\sinh a(1-\xi)}{2\tilde{D}aP_m a(4a^2 - \sinh^2 2a)}\big\{a\sinh a(1-z) \\
& + (\sinh 2a - a\cosh 2a)\sinh a(z+1) + a\sinh 2a\cosh a(z+1)\big\}.
\end{aligned}
$$