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Semigroup ideals with multiplicative semiderivations and commutativity of 3-prime near-rings

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Abstract. In the present paper, we introduce the notion of multiplicative semiderivation and prove some theorems in the setting of a semigroup ideal of a 3-prime near-ring admitting a multiplicative semiderivation. Thereby, it is shown that under appropriate additional hypotheses a near-ring must be a commutative ring. Furthermore, an example is given to illustrate that the 3-primeness hypothesis is not superfluous.

Keywords: prime near-rings, multiplicative derivations, commutativity

MSC 2022 classification: primary 16N60, secondary 16W25, 16Y30

Introduction

Throughout this paper, N is a left near-ring and $Z(N)$ is the multiplicative center of N. For any pair of elements $x, y \in N$, $[x, y]$ denotes the commutator $xy - yx$, while the symbol $x \circ y$ denotes the anticommutator $xy + yx$. Define N to be 3-prime if for all $x, y \in N$, $xNy = \{0\}$ implies $x = 0$ or $y = 0$ and call N 2-torsion free if $(N, +)$ has no elements of order 2. A near-ring N is called a zero-symmetric if $0x = 0$ for all $x \in N$ (recall that left distributivity yields that $x.0 = 0$). A nonempty subset I of N is called a semigroup left ideal (resp. semigroup right ideal) if $NI \subseteq I$ (resp. $IN \subseteq I$) and if I is both

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a semigroup left ideal and a semigroup right ideal of N, then I is said to be semigroup ideal. An additive mapping $d : N \to N$ is said to be a derivation if $d(xy) = xd(y) + d(x)y$ for all $x, y \in N$, or equivalently , as noted in Wang [15], that $d(xy) = d(x)y + xd(y)$ for all $x, y \in N$. An additive mapping $d : N \to N$ is called a semiderivation if there is an additive mapping $g : N \to N$ such that $d(xy) = g(x)d(y) + d(x)y = d(x)y + g(x)d(y)$ for all $x, y \in N$ and d commute with g (see [10]).

Over the last few decades, several authors have investigated the relationship between the commutativity of a near-ring N and certain special types of mappings defined on N. The first result in this direction is due to Bell and Mason [6] who proved that a 2-torsion free 3-prime near-ring N must be a commutative ring if N admits a nontrivial derivation d for which $d(N) \subseteq Z(N)$ or d is a commuting derivation on N. Later, several authors have subsequently refined and extended these results in various directions (see, [2, 3, 7, 8, 9, 10, 14], where further references can be found). In [11], Daif motivated by Martindale in [13] introduced the notion of multiplicative derivation of a ring R as follows: A mapping $d : R \to R$, not necessarily additive, is called a multiplicative derivation if $d(xy) = xd(y)+d(x)y$ holds for all $x, y \in R$. Of course any derivation is a multiplicative derivation, but the converse is not true in general. For more details, see for instance, M. Ashraf et al. [1, Examples 1.1, 1.2] and Z. Bedir et al. [4]. Here, our aim is to investigate some identities with multiplicative semiderivations on some suitable subsets in 3-prime near-rings. Indeed, motivated by the concepts of multiplicative derivation on rings, we initiate the concepts of multiplicative semiderivation on near-rings N as follows: A mapping $d : N \to N$, not necessarily additive, is called a multiplicative semiderivation if there exists a function $g : N \to N$ such that $d(xy) = g(x)d(y) + d(x)y$ for all $x, y \in N$. In case g is the identity map on N, d is of course just a multiplicative derivation on N. Also, any semiderivation is a multiplicative semiderivation, but the generalization is not trivial as the following example shows.

Example 1. Let S be a nonzero zero-symmetric 3-prime left near-ring and let

$$N = \left\{ \begin{pmatrix} 0 & x & y \\ 0 & 0 & 0 \\ 0 & z & 0 \end{pmatrix} \mid 0, x, y, z \in S \right\}.$$

Define maps $d, g : N \to N$ by

$$d\begin{pmatrix} 0 & x & y \\ 0 & 0 & 0 \\ 0 & z & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & xy & 0 \end{pmatrix} \quad \text{and} \quad g\begin{pmatrix} 0 & x & y \\ 0 & 0 & 0 \\ 0 & z & 0 \end{pmatrix} = \begin{pmatrix} 0 & x & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

It can be verified that N is a left near-ring and d is a multiplicative semiderivation on N associated with a function g. But, since d is not an additive map in N, then d cannot be a derivation nor a semiderivation on N.

1 Preliminary results

In this section, we give some well-known results and we add some new lemmas that are very crucial for developing the proofs of our main results. The proofs of the first Lemmas can be found in [5].

Lemma 1. *Let N be a 3-prime near-ring.*

(*i*) [5, Lemma 1.2(iii)] If $z \in Z(N) - \{0\}$ and $xz \in Z(N)$, then $x \in Z(N)$.

(*ii*) [5, Lemma 1.5] If $Z(N)$ contains a nonzero semigroup left ideal or semigroup right ideal, then N is a commutative ring.

Lemma 2. *Let N be a 3-prime near-ring and I be a nonzero semigroup ideal of N.*

(*i*) [5, Lemma 1.4(i)] If $x, y \in N$ and $xIy = \{0\}$, then $x = 0$ or $y = 0$.

(*ii*) [5, Lemma 1.3(i)] If $x \in N$ and $xI = \{0\}$ or $Ix = \{0\}$, then $x = 0$.

Lemma 3. *Let N be a 3-prime near-ring and I be a nonzero semigroup left ideal of N. If N admits a nonzero multiplicative semiderivation d, then $d(I) \neq \{0\}$.*

Proof. Let I be a nonzero semigroup left ideal of N and suppose that $d(I) = \{0\}$. So, we have $d(x) = 0$ for all $x \in I$. Replacing x by yx, where $y \in N$, in the previous relation we get $d(y)x = 0$ for all $y \in N$. Taking tx instead of x, where $t \in N$, in the last equation we obtain $d(y)Nx = \{0\}$ for all $x \in I, y \in N$. Using the fact that $I \neq \{0\}$ and N is 3-prime we conclude that $d(N) = \{0\}$, a contradiction. QED

Lemma 4. *Let N be a 3-prime near-ring admitting a nonzero multiplicative semiderivation d associated with a multiplicative map g on I, then N satisfies the following partial distribution law*

$$(g(x)d(y) + d(x)y)z = g(x)d(y)z + d(x)yz \text{ for all } x, y, z \in I.$$

Proof. Let $x, y, z \in N$. By defining d, comparing the both expressions $d(xyz) = d\big(x(yz)\big)$ and $d(xyz) = d\big((xy)z\big)$, we obtain the required result. QED

Lemma 5. *Let N be a 3-prime near-ring, I be a nonzero semigroup ideal of N and a be an element of I. If N admits a nonzero multiplicative semiderivation d associated with a multiplicative map g on I, then $d(I)a = \{0\}$ implies $a = 0$.*

Proof. According to Lemma 4, the proof can be given by using a similar approach as in the proof of [6, Lemma 3(iii)]. QED

Lemma 6. *Let I be a nonzero semigroup ideal of a near-ring N. N admits a multiplicative semiderivation d associated with a multiplicative map g on I if and only if it is zero-symmetric.*

Proof. Suppose that N admits a multiplicative semiderivation d associated with a map g. In [12, Theorem 1.15] it is proved that N can be expressed as the sum of $N_0 = \{x \in N/0x = 0\}$ the unique maximal zero-symmetric subnear-ring of N, and $N_c = 0N = \{0x/x \in N\}$ the unique maximal constant subnear-ring of N. Furthermore, as a group, N semidirect product of N_0 and N_c, i.e. N_0 is a normal subgroup of N, $N = N_0 + N_c$ and $N_0 \cap N_c = \{0\}$. Now, let z an arbitrary element of N_c. By defining N_c, there is an element $x \in N$ such that $z = 0x$. Hence, z satisfies the following properties: $0z = 0.0x = z$, $z^2 = 0x.0x = 0(x.0)x = 0x = z$ and $z \in I$. Furthermore,

$$d(z) = d(z^2) = g(z)d(z) + d(z)z = g(z)d(z) + d(z).(0.z) = g(z)d(z) + z.$$

Multiplying left-hand side of the latter relation by $g(z)$, we find that

$$\begin{aligned} g(z)d(z) &= g(z)d(z) + g(z).z \\ &= g(z)d(z) + g(z)(0.z) \\ &= g(z)d(z) + (g(z).0)z \\ &= g(z)d(z) + 0.z \\ &= g(z)d(z) + z \end{aligned}$$

then $z = 0$. Consequently, N reduces to N_0 and therefore N is a zero-symmetric near-ring. Conversely, suppose that N is zero-symmetric. Clearly, the zero map on N is a multiplicative semiderivation of N. QED

2 Main results

In [6], H. E. Bell and G. Mason show that a prime near-ring N must be commutative if it admits a derivation d which satisfies $d(N) \subseteq Z(N)$. After, A. Raji et al. in [8] proved the same result in the case where d is a generalized semiderivation. The fundamental purpose of the following theorem is to establish a more precise version of the above result using fewer assumptions. In fact, we have obtained the following:

Theorem 1. *Let N be a 3-prime near-ring and I be a nonzero semigroup ideal of N. If N admits a nonzero multiplicative semiderivation d associated with a multiplicative map g on I such that $d(I) \subseteq Z(N)$, then N is a commutative ring.*

Proof. By the hypothesis given, we have

$$d(xy)y = yd(xy) \text{ for all } x, y \in I.$$

Using Lemma 4, we get

$$g(x)d(y)y + d(x)yy = yg(x)d(y) + yd(x)y \text{ for all } x, y \in I,$$

this implies that

$$g(x)d(y)y = yg(x)d(y) \text{ for all } x, y \in I.$$

Which can be rewritten as

$$d(y)N[g(x), y] = \{0\} \text{ for all } x, y \in I.$$

In view of 3-primeness of N, the latter relation gives that

$$d(y) = 0 \text{ or } g(x)y = yg(x) \text{ for all } x, y \in I. \tag{2.1}$$

Let y an arbitrary element of I such that $d(y) = 0$. So, $d(xy) = d(x)y \in Z(N)$ for all $x \in I$. By Lemma 1(i), the last expression assures that

$$\text{either } d(x) = 0 \text{ for all } x \in I \text{ or } y \in Z(N). \tag{2.2}$$

Since by Lemma 3, $d(I) \neq \{0\}$, (2.2) forces $y \in Z(N)$. Hence, (2.1) shows that

$$g(x)y = yg(x) \text{ for all } x, y \in I. \tag{2.3}$$

Now, let $x, y, t \in I$, we have $d(xt)y = yd(xt)$. Using Lemma 4 together equation (2.3) and after simplifying, the preceding equation shows that

$$d(x)yt = d(x)ty \text{ for all } x, y, t \in I,$$

which leads to

$$d(x)N[t, y] = \{0\} \text{ for all } x, y, t \in I.$$

Using the 3-primeness and the fact that $d(I) \neq \{0\}$, we get $yt = ty$ for all $y, t \in I$, which easily gives $I \subseteq Z(N)$. By Lemma 1(ii) we conclude that N is a commutative ring. Thus our proof is complete. QED

In [2, Corollary 4.1] M. Ashraf and S. Ali studied commutativity in prime near-rings with a nonzero derivation d for which $d([x,y]) = 0$ for all $x, y \in N$. Thereby, A. Raji et al. in [9, Theorem 3] proved the same result in the setting of a semigroup ideal of a 3-prime near-ring admitting a semiderivation. Our second theorem treats the above result in a more general situation.

Theorem 2. *Let N be a 3-prime near-ring and I be a nonzero semigroup ideal of N. If N admits a nonzero multiplicative semiderivation d associated with a multiplicative map g on I such that $d([x,y]) = 0$ for all $x, y \in I$, then N is a commutative ring.*

Proof. Suppose that there is d such that

$$d([x,y]) = 0 \text{ for all } x, y \in I. \tag{2.4}$$

Taking xy instead of y in (2.4) and noting that $[x, xy] = x[x,y]$, we get

$$d(x[x,y]) = d(x)[x,y] = 0 \text{ for all } x, y \in I,$$

so that

$$d(x)xy = d(x)yx \text{ for all } x, y \in I. \tag{2.5}$$

Writing yt for y in (2.5), where $t \in N$, and using (2.5) we obtain

$$d(x)yxt = d(x)ytx \text{ for all } x, y \in I, t \in N,$$

it follows that

$$d(x)I[x,t] = \{0\} \text{ for all } x \in I, t \in N. \tag{2.6}$$

Applying Lemma 2(i), (2.6) shows that

$$d(x) = 0 \text{ or } x \in Z(N) \text{ for all } x \in I. \tag{2.7}$$

Let x an element fixed in I and suppose that $d(x) = 0$. Our goal to show that $x \in Z(N)$. Indeed, if $x = 0$ then $x \in Z(N)$; else i.e $x \neq 0$, in this case there exists y_0 in I such that $d(y_0x) \neq 0$, since otherwise if $d(yx) = 0$ for all $y \in I$, we get $d(y)x = 0$ for all $y \in I$ and therefore $x = 0$ by Lemma 5, which is a contrary to our hypothesis. Now, since $d(y_0x) \neq 0$ and $y_0x \in I$, then (2.7) assures that $y_0x \in Z(N)$. Putting y_0x^2 instead of x in (2.4), we obtain $d(y_0x)[x,y] = 0$ for all $y \in I$. Replacing y by yt, where $t \in N$, in the last equation, we arrive at

$$d(y_0x)I[x,t] = \{0\} \text{ for all } t \in N. \tag{2.8}$$

As $d(y_0x) \neq 0$, according to Lemma 2(i), equation (2.8) yields $x \in Z(N)$. Consequently, we see that $I \subseteq Z(N)$ and therefore N is a commutative ring by Lemma 1(ii). QED

As an application of Theorem 1, we get the following results.

Theorem 3. *Let N be a 3-prime near-ring and I be a nonzero semigroup ideal of N. If N admits a nonzero multiplicative semiderivation d associated with a multiplicative map g on I such that $[d(x), y] \in Z(N)$ for all $x, y \in I$, then N is a commutative ring.*

Proof. Assume that $[d(x), y] \in Z(N)$ for all $x, y \in I$. Substituting $d(x)y$ for y in the preceding expression, we obtain that

$$d(x)[d(x), y] \in Z(N) \text{ for all } x, y \in I,$$

which, because of Lemma 1(i), implies that

$$[d(x), y] = 0 \text{ or } d(x) \in Z(N) \text{ for all } x, y \in I. \tag{2.9}$$

Suppose that there is an element $x \in I$ satisfies $[d(x), y] = 0$ for all $y \in I$, then $d(x)y = yd(x)$ for all $y \in I$. Taking yt instead of y, where $t \in N$, in the last equation and using it, we arrive at $I[d(x), t] = \{0\}$ for all $t \in N$. And therefore, $d(x) \in Z(N)$ by Lemma 2(ii). Consequently, (2.9) reduces to $d(I) \subseteq Z(N)$. According to Theorem 1, we obtain the conclusion that N is a commutative ring. QED

As an application of the previous theorems, we can get the following corollary if d acts as a nonzero derivation or a nonzero multiplicative derivation of N or a nonzero semiderivation of N associated with a multiplicative map g on I, where I is a nonzero semigroup ideal of N.

Corollary 1. *Let N be a 3-prime near-ring and let I be a nonzero semigroup ideal of N. Then the following assertions are equivalent*

1. *$d(I) \subseteq Z(N)$,*
2. *$d([x, y]) = 0$ for all $x, y \in I$,*
3. *$[d(x), y] \in Z(N)$ for all $x, y \in I$,*
4. *N is a commutative ring.*

Theorem 4. *Let N be a 3-prime near-ring and I be a nonzero semigroup ideal of N. If N admits a nonzero multiplicative semiderivation d associated with a multiplicative map g on I such that $d(x) \circ y \in Z(N)$ for all $x \in I$, $y \in N$, then N is a commutative ring.*

Proof. Assume that

$$d(x) \circ y \in Z(N) \text{ for all } x \in I,\ y \in N. \tag{2.10}$$

Suppose that $Z(N) = \{0\}$. In this case the relation (2.10) leading to $d(x) \circ y = 0$ for all $x \in I, y \in N$; in such a way $d(x)y = -yd(x)$ for all $x \in I$, $y \in N$.

Substituting yt for y, where $t \in N$, in the preceding equation and using it again, we find that $(-yd(x))t = -ytd(x)$ for all $x \in I$, $y, t \in N$. This identity shows that

$$y(-d(x))t = yt(-d(x)) \text{ for all } x \in I, y, t \in N.$$

Accordingly,

$$N[-d(x), t] = 0 \text{ for all } x \in I, t \in N.$$

Invoking lemma 2(ii), it follows that $-d(x) \in Z(N) = \{0\}$ for all $x \in I$ which means that $d(I) = \{0\}$. According to Lemma 3, the last result implies that $d = 0$ which is contrary to our hypothesis. And therefore, $Z(N) \neq \{0\}$. Now, choosing $z_0 \in Z(N) - \{0\}$ and putting z_0 in the place of y in (2.10), we get $d(x) \circ z_0 \in Z(N)$ for all $x \in I$ which leads to $z_0(d(x) + d(x)) \in Z(N)$ for all $x \in I$. Taking into account Lemma 1(i), we obtain $d(x) + d(x) \in Z(N)$ for all $x \in I$. Then, replacing y by $d(x)$ in (2.10) and using the fact that $d(x) \circ d(x) = d(x)(d(x) + d(x))$, we get

$$d(x)(d(x) + d(x)) \in Z(N) \text{ for all } x \in I. \tag{2.11}$$

As $(d(x) + d(x)) \in Z(N)$, according to Lemma 1(i), (2.11) shows that either

$$d(x) + d(x) = 0 \text{ or } d(x) \in Z(N) \text{ for all } x \in I. \tag{2.12}$$

We may fix an element $x \in I$ such that $d(x) + d(x) = 0$, then $d(x) = -d(x)$. Taking $d(x)y$ instead of y in (2.10), we obtain $d(x)(d(x) \circ y) \in Z(N)$ for all $y \in N$. In light of Lemma 1(i), the latter identity gives

$$d(x) \circ y = 0 \text{ or } d(x) \in Z(N) \text{ for all } y \in N. \tag{2.13}$$

If $d(x) \circ y = 0$ for all $y \in N$, we have $d(x)y = -yd(x) = y(-d(x)) = yd(x)$ for all $y \in N$ which means that $d(x) \in Z(N)$. Consequently, (2.13) reduces to $d(x) \in Z(N)$ and also (2.12) reduces to $d(x) \in Z(N)$ for all $x \in I$. By virtue of Theorem 1, N is a commutative ring. This completes the proof of our Theorem. QED

Theorem 5. *Let N be a 2-torsion free 3-prime near-ring and I be a nonzero semigroup ideal of N. If N admits a nonzero multiplicative semiderivation d associated with a multiplicative map g on I such that $x \circ d(y) \in Z(N)$ for all $x \in N$, $y \in I$, then N is a commutative ring.*

Proof. By hypothesis given, we have

$$x \circ d(y) \in Z(N) \text{ for all } x \in N, y \in I. \tag{2.14}$$

Assume that $Z(N) = \{0\}$. Substituting $d(y)$ for x in (2.14), we find that $2d(y)d(y) = 0$ for all $y \in I$. In view of 2-torsion freeness, we conclude that

$d(y)d(y) = 0$ for all $y \in I$. On the other hand, (2.14) yields $xd(y) + d(y)x = 0$ for all $x \in N, y \in I$. Left multiplying the last result by $d(y)$, we get $d(y)xd(y) + d(y)d(y)x = 0$ for all $x \in N$, $y \in I$. Since, because of Lemma 6, N is a zero-symmetric near-ring, the preceding identity shows that $d(y)xd(y) = 0$ for all $x \in N$, $y \in I$ and whence it follows that $d(y)Nd(y) = \{0\}$ for all $y \in I$. In the light of the 3-primeness of N we conclude $d(I) = \{0\}$, a contradiction and therefore $Z(N) \neq \{0\}$. Taking $0 \neq z_0 \in Z(N)$ and replacing x by z_0 in (2.14), we get

$$z_0(d(y) + d(y)) \in Z(N) \text{ for all } y \in I. \tag{2.15}$$

Invoking Lemma 1(i) and as $z_0 \neq 0$, (2.15) shows that $d(y)+d(y) \in Z(N)$ for all $y \in I$. Now, replacing x by $d(y)$ in (2.14), we find that $d(y)(d(y)+d(y)) \in Z(N)$ for all $y \in I$. Using Lemma 1(i), the former identity yields

$$d(y) + d(y) = 0 \text{ or } d(y) \in Z(N) \text{ for all } y \in I. \tag{2.16}$$

By the 2-torsion freeness, the first condition of (2.16) gives $d(y) = 0$ and therefore (2.16) reduces to $d(I) \subseteq Z(N)$, then N is a commutative ring by Theorem 1. QED

The following example demonstrates that the 3-primeness assumption is essential in Lemma 3 and in the hypotheses of the our theorems.

Example 2. Let S, N, d, g be defined as in Example 1, and let us define I by:

$$I = \left\{ \begin{pmatrix} 0 & x & 0 \\ 0 & 0 & 0 \\ 0 & z & 0 \end{pmatrix} \mid 0, x, z \in S \right\}.$$

It is easy to verify that N is a non 3-prime near-ring and I is a nonzero semigroup ideal of N. Moreover, d is a nonzero multiplicative semiderivation which satisfies the properties:

1. $d(I) = \{0\}$ and $d(N) \neq \{0\}$,
2. $d(I) \subseteq Z(N)$,
3. $d([A, B]) = 0$ for all $A, B \in I$,
4. $[d(A), B] \in Z(N)$ for all $A, B \in I$,
5. $d(A) \circ B \in Z(N)$ for all $A \in I, B \in N$,
6. $A \circ d(B) \in Z(N)$ for all $A \in N, B \in I$.

However, N is not a commutative ring.

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