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Relative nullity foliation of the screen distribution of lightlike Einstein hypersurfaces in Lorentzian spaces

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Abstract. This paper deals with results concerning the relative nullity foliation of the screen distribution of a lightlike Einstein hypersurface M in the Lorentzian space $\mathbb{R}_1^{n+2}$ and gives a characterization theorem for the relative nullity spaces. Many differences from the Riemannian case are due to the fact that the metric in consideration is degenerate.

Keywords: lightlike Einstein hypersurfaces, relative nullity foliation, screen distribution, index of relative nullity

MSC 2000 classification: primary 53C50, secondary 53C25

1 Introduction and result

We investigate in this paper the relative nullity foliation of the screen distribution of a lightlike Einstein hypersurface M in $\mathbb{R}_1^{n+2}$. Our result stands as follows

1 Theorem. *Let $\overline{M}$ be the Lorentzian space $\mathbb{R}_1^{n+2}(n \geq 3)$ and $(M, g, S(TM))$ a lightlike-Einstein hypersurface in $\overline{M}$. Then:*

(1) The relative nullity space distribution T^{*0} of the screen distribution is smooth and involutive on any subset with constant index of relative nullity.

(2) The set G of points in M where $\nu(x) = \nu_0$ is open in M.

(3) The foliation T^{*0} of the screen distribution is totally geodesic in M and $\mathbb{R}_1^{n+2}$

(4) If M is not Ricci flat,

a. T^{*0} is an isotropic distribution along M.

b. and if the shape operators A_ξ^* and A_N are simultaneously diagonalizable on M, then $T^{*0} = TM^\perp$.

This main result is similar to that of the indefinite isometric immersion case. However, there are non trivial differences arising in the last part of our theorem. In the next two paragraphs, we summarize basic formulae concerning geometric objects on lightlike submanifolds and lightlike Einstein hypersurfaces, using notations of [3], and basic properties of the relative nullity space of a lightlike Einstein hypersurface. The last part of the paper is the proof of the main theorem.

2 Preliminaries and basic facts

The fundamental difference between the theory of lightlike (or degenerate) submanifolds (M^n, g), and the classical theory of submanifolds of a semi-Riemannian manifold $(\bar{M}^{n+p}, \bar{g})$ comes from the fact that in the first case, the normal vector bundle $TM^\perp$ intersects with the tangent bundle TM in a non zero subbundle, denoted $\mathrm{Rad}(TM)$, so that

$$\mathrm{Rad}(TM) = TM \cap TM^\perp \neq \{\, 0 \,\} \tag{1}$$

Given an integer $r > 0$, the submanifold M is said to be r-lightlike (or r-degenerate) if the rank of $\mathrm{Rad}(TM)$ is equal to r everywhere.

In particular, lightlike hypersurfaces of Lorentzian spaces, have their degenerate metric of signature $(0, + \dots, +)$. Then the induced metric on their screen distribution $S(TM)$ is non degenerate and positive definite metric (see [3]). In this case, relation (1) becomes $\mathrm{Rad}(TM) = TM^\perp$ and we have the following splitting in an orthogonal direct sum

$$TM = S(TM) \perp \mathrm{Rad}(TM). \tag{2}$$

Throughout the paper, we will consider integrable screen distributions $S(TM)$, that is at each point $p \in M$, there is a submanifold (a leaf) $S \subset M$ such that $T_pS = S(T_pM)$ and for a vector bundle E, $\Gamma(E)$ will denoted the space of smooth sections over E, and we also use $\mathcal{F}(\overline{M})$ to denote the space of smooth functions on $\overline{M}$.

We have the following normalization result.

2 Theorem (Duggal-Bejancu [3], p. 79)**.** *Let $(M, g, S(TM))$ be a lightlike hypersurface of a semi-Riemannian manifold $(\overline{M}, \overline{g})$. Then there exists a unique vector bundle* $\mathrm{tr}(TM)$ *of rank* 1 *over M, such that for any non-zero section ξ of $TM^{\perp}$ on a coordinate neighbourhood $\mathcal{U} \subset M$, there exists a unique section N of* $\mathrm{tr}(TM)$ *on $\mathcal{U}$ satisfying*

$$\overline{g}(\xi, N) = 1, \quad \overline{g}(N, N) = \overline{g}(N, W) = 0, \quad \forall\, W \in \Gamma(S(TM|_{\mathcal{U}})). \tag{3}$$

Consider $\overline{\nabla}$ the Levi-Civita connection of $(\overline{M}, \overline{g})$ and ∇ the induced connection on the lightlike hypersurface (M, g). The connection ∇ on (M, g) is not unique in general. It depends on both g and $S(TM)$, and is associate to the triplet $(M, g, S(TM))$. One can show that it is independent of $S(TM)$ if and only if the second fundamental form h of M (defined in (5) below) vanishes identically (see [3, Theorem 2.1, p. 87]).

With the decompositions in orthogonal direct sums (2) and

$$T\overline{M}|_M \quad = \quad S(TM) \perp (TM^{\perp} \oplus \mathrm{tr}(TM)) = TM \oplus \mathrm{tr}(TM), \tag{4}$$

Gauss and Weingarten formulae can been written

$$\overline{\nabla}_X Y = \nabla_X Y + h(X, Y), \quad \forall\, X, Y \in \Gamma(TM), \tag{5}$$

$$\overline{\nabla}_V X = -A_V X + \nabla^t_X, \quad \forall\, X \in \Gamma(TM) \quad \forall\, V \in \Gamma(\mathrm{tr}(TM)), \tag{6}$$

where $\nabla_X Y$ and $A_V X$ belong to $\Gamma(TM)$, while h is a $\Gamma(\mathrm{tr}(TM))-$valued symmetric $\mathcal{F}(M)$-bilinear form on $\Gamma(TM)$, A_V is an $\mathcal{F}(M)$-linear operator on $\Gamma(TM)$ and ∇^t is a linear connection on the lightlike transversal vector bundle $\mathrm{tr}(TM)$.

Define a symmetric $\mathcal{F}\left(\mathcal{U}\right)$-bilinear form B and a 1-form τ on the coordinate neighbourhood $\mathcal{U}$ by

$$B(X, Y) = \bar{g}\left(h(X, Y), \xi\right), \quad \forall\, X, Y \in \Gamma\left(TM|_{\mathcal{U}}\right), \tag{7}$$

$$\tau(X) = \bar{g}\left(\nabla^t_X N, \xi\right), \quad \forall\, X \in \Gamma\left(TM|_{\mathcal{U}}\right). \tag{8}$$

The 1-form τ depends on the vector field ξ and it's easy to see that if $\bar{\xi} = \alpha\xi$ with α a positive smooth function on M, the associated 1-form $\bar{\tau}$ is related to τ by

$$\tau(X) = \bar{\tau}(X) + X\left(Log\alpha\right), \quad \forall\, X \in \Gamma\left(TM|_{\mathcal{U}}\right). \tag{9}$$

The induced connection ∇ on a lightlike hypersurface M is not metric in general and the Ricci tensor associated is not symmetric, contrary to the case of semi-Riemannian manifolds. However, for lightlike Einstein hypersurfaces, due to the symmetry of the induced degenerate metric g, the Ricci tensor is symmetric, and the notion of Einstein manifold doesn't depend on the choice of the screen distribution $ST(M)$. Consequently

3 Proposition. *On a lightlike-Einstein hypersurface the* $1-$*form* τ *in (8) is closed.*

Proof. Define Ricc as

$$\mathrm{Ricc}(X,Y) = \mathrm{trace}(Z \longrightarrow R(Z,X)Y),\ \forall X,Y \in \Gamma(TM),$$

where R denotes the Riemann tensor of the induced connection ∇ on M.

Consider a local quasi-orthonormal frame-field $\{X_0, N, X_i\}_{i=1,\dots,n}$ on $\overline{M}$ where $\{X_0, X_i\}$ is a local frame-field on M with respect to the decomposition (4) with N, the unique section of transversal bundle $\mathrm{tr}(TM)$ satisfying (3), and $\xi = X_0$. Put $R_{ls} := \mathrm{Ricc}(X_s, X_l)$ and $R_{0k} := \mathrm{Ricc}(X_k, X_0)$. A direct computation using the frame-field $\{X_0, N, X_i\}$ gives locally

$$R_{ls} - R_{sl} = 2d\tau(X_l, X_s),$$
$$R_{0k} - R_{k0} = 2d\tau(X_0, X_k).$$

Consequently, because The Ricci tensor is symmetric on M which is Einstein, we have $d\tau = 0$. $\boxed{QED}$

We also have

4 Proposition. *If* $(M, g, S(TM))$ *is a lightlike-Einstein hypersurface, there exists, on all coordinate neighbourhood* $\mathcal{U}$*, a pair* $\{\xi, N\}$ *such that the* $1-$*form* τ *in (8) vanishes identically.*

Proof. From proposition 3 τ is closed. Poincare lemma implies locally on $\mathcal{U}$, $\tau = d\zeta$ for some function $\zeta \in \mathcal{F}(\mathcal{U})$ that is

$$\tau(X) = X \cdot (\zeta).$$

Using relation (9), for $\alpha = \exp(\zeta)$ yields

$$\tau(X) = \bar{\tau}(X) + X \cdot \zeta = \bar{\tau}(X) + \tau(X),$$

so $\bar{\tau}(X) = 0$ for all $X \in \Gamma(TM|_{\mathcal{U}})$. Then, taking $\bar{\xi} = \exp(\zeta)\xi$, one obtains $\bar{\tau} \equiv 0$ on $\mathcal{U}$. The corresponding $\bar{N}$ is $\overline{N} = (1/\exp(\zeta))\,N$. $\boxed{QED}$

For the sake of simplicity we also denote this pair by $\{\xi, N\}$. Then, relation (6) may be written as

$$\bar{\nabla}_X N = -A_N X, \quad \forall X \in \Gamma(TM|_{\mathcal{U}}). \tag{10}$$

For the sake of future use, we also have

5 Proposition. *Let $(M, g, S(TM))$ be a lightlike hypersurface of the Lorentz space $\mathbb{R}_1^{n+2}(n \geq 3)$. Then*

$$\mathrm{Ricc}(X,Y) = g(A_\xi^* X, Y) tr A_N - g(A_N X, A_\xi^* Y), \quad \forall\, X, Y \in \Gamma(TM|_{\mathcal{U}}).$$

Proof. We have

$$\begin{aligned} \mathrm{Ricc}(X,Y) &= \mathrm{trace}\,(Z \mapsto R(Z,X)Y) \\ &= \sum g\,(R(W_i,X)Y,W_i) + \bar{g}\,(R(\xi,X)Y,N)\,. \end{aligned}$$

The Riemann tensor $\bar{R}$ of $\mathbb{R}_1^{n+1}$ is zero and from Gauss and Codazzi equation, we obtain

$$g\,(R(W_i,X)Y,W_i) = B(X,Y)C(W,W_i) - B(W,Y)C(X,W_i).$$

In other side, from the equality

$$\bar{g}\left(\bar{R}(X,Y)Z,N\right) = \bar{g}\,(R(X,Y)Z,N)\,, \quad \forall\, X, Y \in \Gamma(TM),$$

we obtain for $\bar{R} = 0$, $\bar{g}\,(R(\xi,X)Y,N) = 0$ and

$$\mathrm{Ricc}(X,Y) = B(X,Y)\left(\sum_{i=1}^{n} C(W_i,W_i)\right) - \left(\sum_{i=1}^{n} B(W_i,Y)C(X,W_i)\right).$$

But $C(W_i,W_i) = g(A_N W_i, W_i)$ and $\bar{g}\,(A_N \xi, N) = 0$, hence

$$\sum_{i=1}^{n} C(W_i,W_i) = \mathrm{trace}\, A_N.$$

And the result holds using relations

$$B(W_i,Y) = g(A_\xi Y, W_i) \quad \text{and} \quad C(X,W_i) = g(A_N X, W_i).$$

$\boxed{QED}$

3 The relative nullity distribution of the screen distribution

Let M be a lightlike hypersurface in the Lorentzian space $\mathbb{R}_1^{n+2}$. The relative nullity space at a point x is defined by:

$$T^{*0}(x) = \{\, X \in T_x M / A_\xi^* X = 0,\ \forall\, \xi \in T_x M^\perp \,\}$$

where A^*_ξ is the shape operator of M. This distribution characterize somehow the totally geodesic property of M; indeed $T^{*0}(x) = T_x(M)$ is equivalent to M is totally geodesic.
The dimension $\nu(x)$ of $T^{*0}(x)$ is called the index of relative nullity at x. The value $\nu_0 = \min_{x\in M} \nu(x)$ is called the index of minimum relative nullity.

Nullity spaces in Riemannian geometry have been studied by many authors ([4, p. 68], [5] and references therein). Abe and Magid (see [1]) have extended the study of the relative nullity foliation to isometric immersion between manifolds with indefinite metrics. In the case of lightlike isotropic submanifolds M of semi-Riemannian manifolds, the first transverse space at a point x define by

$$T_1(x) = \mathrm{span}\{\, h^s(X,Y),\ X,Y \in \Gamma(T_xM)\,\}$$

where h^s is defined from Gauss formula (5), has been used by C. Atindogbe and *al* in [2] to the reduction of the codimension of an isotropic immersion. It is worth noticing here that as for the relative nullity space, $T_1(x) = \{\,0\,\}$ for all $x \in M$ is equivalent to M is totally geodesic in the semi-Riemannian manifold $\overline{M}^{n+p}$.

For the proof of the main theorem of this paper, we need the following characterization of the relative nullity space. We have

6 Proposition.

$$T^{*0}(x) = \{\, X \in T_xM,\ h(X,PY) = 0,\ \forall\, Y \in T_xM\,\}$$

where P is the projection morphism of $\Gamma\,(TM)$ *on* $\Gamma\,(ST(M))$

Proof. We have

$$\begin{aligned} X \in T^{*0}(x) &\iff A^*_\xi X = 0, \quad \forall\, \xi \in T_xM^\perp \\ &\iff g(A^*_\xi X, PY) = 0, \quad \forall\, Y \in T_xM, \quad \forall \xi \in T_xM^\perp \\ &\iff h(X,PY) = 0, \quad \forall\, Y \in T_xM, \quad \forall \xi \in T_xM^\perp. \end{aligned}$$

$\boxed{QED}$

7 Remark. Due to the fact that $A^*_\xi \xi = 0$ one has

$$\dim T^{*0}(x) \geq 1, \quad \forall\, x \in M$$

Moreover

$$T^{*0}(x) \supset T_xM^\perp \tag{11}$$

Therefore, T^{*0} is a degenerate distribution along M.

4 Proof of the main theorem

(i) Let Ω be an open subset of M on which the relative nullity index is a constant ν, and $x_0 \in \Omega$. From (11), we have

$$T^{*0}(x_0) = P(T^{*0}(x_0)) \perp T_{x_0}M^{\perp}$$

We claim that

$$T^{*0\perp}(x_0) = \mathrm{span}\{\, A^*_\xi Y,\ Y \in T_{x_0}M,\ \xi \in T_{x_0}M^{\perp}\,\} \perp T_{x_0}M^{\perp}.$$

Let $\perp_S$ denote the orthogonality symbol in $S(TM)$. For $Y \in T_{x_0}M,\ \xi \in T_{x_0}M^{\perp}$ and $X \in P(T^{*0}(x_0))$, we have

$$g(A^*_\xi Y, X) = g(Y, A^*_\xi X) = 0,$$

and

$$\mathrm{span}\{\, A^*_\xi Y\,\} \subset P(T^{*0}(x_0))^{\perp_S}.$$

Let $Z \in span\{\, A^*_\xi Y\,\}^{\perp_S}$ and $Y \in T_{x_0}M$, we have

$$0 = g(Z, A^*_\xi Y) = g(A^*_\xi Z, Y), \quad \forall\, Y \in T_{x_0}M.$$

Then

$$A^*_\xi Z \in S(T_{x_0}M) \cap T_{x_0}M^{\perp}0 = \{\,0\,\}.$$

That is $A^*_\xi Z = 0$ and $Z \in P(T^{*0}(x_0))$. Hence

$$\mathrm{span}\{\, A^*_\xi Y\,\}^{\perp_S} \in P(T^{*0}(x_0)) \text{ and } P(T^{*0}(x_0))^{\perp_S} \subset \mathrm{span}\{\, A^*_\xi Y\,\}.$$

We conclude that

$$P(T^{*0}(x_0))^{\perp_S} = \mathrm{span}\{\, A^*_\xi Y\,\} \text{ and } T^{*0\perp}(x_0) = \mathrm{span}\{\, A^*_\xi Y\,\} \perp T_{x_0}M^{\perp}.$$

There exist vector fields $Y_1, \ldots, Y_{n-\nu+1} \in T_{x_0}M$ such that

$$\{\, \xi(x_0), A^*_{\xi(x_0)}Y_1, \ldots, A^*_{\xi(x_0)}Y_{n-\nu+1}\,\}$$

represent a basis of $T^{*0}(x_0)^{\perp}$. Take smooth local extensions of $\xi(x_0)$ and $Y_1, \ldots, Y_{n-\nu+1} \in T_{x_0}M$ in $TM^{\perp}$ and TM respectively. By continuity, the vector fields $\{\, \xi(x_0), Y_1, \ldots, Y_{n-\nu+1}\,\}$ remain linearly independent in a neighbourhood $\mathcal{W} \subset \Omega$ of x_0, and then $T^{*0\perp}$ is a smooth distribution. Consequently, T^{*0} is a smooth distribution.

(ii) From the arguments developed in (i) it is obvious that G is open.

(iii) From Gauss-Codazzi equations, for all $U \in TM^{\perp}$ and $X, Y, Z \in \Gamma(TM)$, we have

$$\overline{g}(\bar{R}(X,Y)Z, U) = \overline{g}((\nabla_X h)(Y,Z) - (\nabla_Y h)(X,Z), U).$$

Assume $X \in \Gamma(TM)$ and $Y, Z \in T^{*0}(x)$. Then,

$$(\nabla_X h)(Y,Z) = \nabla^t_X h(Y,Z) - h(\nabla_X Y, Z) - h(Y, \nabla_X Z),$$

and

$$\begin{aligned}(\nabla_X h)(Y,Z) - (\nabla_Y h)(X,Z) = \nabla^t_X h(Y,Z) - \nabla^t_Y h(X,Z) + h(\nabla_Y X, Z)\\ + h(X, \nabla_Y Z) - h(\nabla_X Y, Z) - h(Y, \nabla_X Z).\end{aligned}$$

But

$$g(h(X, PY), \xi) = g(A^*_\xi X, PY),$$

and

$$\tau(X) = \overline{g}(\nabla^t_X N, \xi) = 0.$$

So

$$\overline{g}(\nabla^t_X h(Y,Z), \xi) = \overline{g}(\nabla^t_X B(Y,Z)N, \xi) = X \cdot B(Y,Z).$$

Therefore

$$\begin{aligned}X \cdot B(Y,Z) - Y \cdot B(X,Z) + \overline{g}(h(\nabla_Y X, Z)\\ + h(X, \nabla_Y Z) - h(\nabla_X Y, Z) - h(Y, \nabla_X Z), \xi) = 0.\end{aligned}$$

We have

$$\begin{aligned}X &= PX + \overline{g}(X,N)\xi\\ Y &= PY + \overline{g}(Y,N)\xi.\end{aligned}$$

Then, from (7), using the fact that $B(X,\xi) = 0$, $\forall\, X \in \Gamma\,(TM|_{\mathcal{U}})$ and Proposition 6 we have

$$B(Y,Z) = \overline{g}(h(Z,PY),\xi) + \overline{g}(Y,N)B(\xi,Z) = 0.$$

Similarly

$$B(X,Z) = 0.$$

On the other side, we have

$$\begin{aligned}h(\nabla_Y X, Z) &= h(\nabla_Y PX + \overline{g}(X,N)\xi, Z)\\ &= h(\nabla^*_Y PX + h^*(Y,PX) + \overline{g}(X,N)\xi, Z)\\ &= h(\nabla^*_Y PX, Z) = 0,\ (\text{for } Z \in T^{*0}(x) \text{ and } \nabla^*_Y PX \in S(TM)).\end{aligned}$$

Also

$$h(\nabla_X Y, Z) = 0.$$

Then

$$\overline{g}(h(X, \nabla_Y Z) - h(Y, \nabla_X Z), \xi) = 0.$$

But

$$\begin{aligned} h(Y, \nabla_X Z) &= h(Y, \nabla_X (PZ + \overline{g}(Z, N)\xi)) \\ &= h(Y, \nabla_X PZ) + \overline{g}(Z, N) h(Y, \nabla_X \xi) \\ &= h(Y, \nabla^*_X PZ + h^*(X, PZ)) + \overline{g}(Z, N) h(Y, -A^*_\xi X - \tau(X)\xi) \\ &= 0. \end{aligned}$$

So $h(Y, \nabla_X Z) = 0$. Hence

$$\overline{g}(h(X, \nabla_Y Z), \xi) = 0, \quad \forall\, X \in \Gamma(TM),$$

that is

$$h(\nabla_Y Z, PX) = 0, \quad \forall\, X \in \Gamma(TM).$$

From Proposition 6 we deduce that $\nabla_Y Z \in T^{*0}(x)$.

We conclude that T^{*0} is totally geodesic in TM and $\mathbb{R}^{n+2}_1$ and (iii) is proved.

(iv) We assume now that M is not Ricci flat.

(a) From Proposition 5 we have

$$g(A^*_\xi X, Y) tr A_N - g(A_N X, A^*_\xi Y) - \rho g(X, Y) = 0, \quad \forall\, X, Y \in \Gamma(TM|_{\mathcal{U}})$$

But

$$X, Y \in T^{*0}(x) \Longrightarrow A^*_\xi X = A^*_\xi Y = 0,$$

and the hypothesis implies $\rho \neq 0$. So

$$g(X, Y) = 0, \quad \forall\, X, Y \in T^{*0}(x)$$

that is the distribution T^{*0} is isotropic along M. We infer that T^* is of rank one, for there is no isotropic distribution in $\mathbb{R}^{n+1}_1$ of which the rank is larger than one.

(b) Now, take $X \in T^{*0}(x)$ and $Y \in T_x M$

$$g(A^*_\xi X, Y) tr A_N - g(A_N X, A^*_\xi Y) - \rho g(X, Y) = 0,$$

so

$$g(A^*_\xi X, Y) tr A_N - g(A^*_\xi A_N X, Y) - \rho g(X, Y) = 0.$$

If the shape operators A^*_ξ and A^*_N are simultaneously diagonalizable at $x \in M$, then they commute $A^*_\xi A_N = A_N A^*_\xi$.
Consequently

$$g(A^*_\xi X, Y) tr A_N - g(A_N A^*_\xi X, Y) - \rho g(X, Y) = 0$$

that is

$$g(X, Y) = 0, \text{ for } A^*_\xi X = 0 \text{ and } \rho \neq 0.$$

So

$$X \in T^{*0}(x) \text{ implies } g(X, Y) = 0, \quad \forall Y \in T_x M.$$

Then $X \in T_x M^\perp$ and we deduce that $T^{*0}(x) \subset T_x M^\perp$.

From (11) we conclude that

$$T^{*0}(x) \equiv T_x M^\perp.$$

$\boxed{QED}$

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