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Neutrosophic Exponentiated Power Lomax Distribution

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The probability distribution is of great significance in probability theory, which is inherent in virtually all the branches of science. It is said to be used selectively in actuarial science with reference to insurance and finance, medicine, agriculture, demography and econometrics. However, the main contribution of the current research work is to propose a new distribution called as neutrosophic exponentiated power Lomax distribution or briefly NEPL. Several other mathematical characteristics that describe life survival and the related characteristics, such as hazard rate and functions and moment-generating functions and other tests of mean, variance, and standard deviation, asymmetry and kurtosis, have been built and analyzed. Monte Carlo method has been applied also to assess the efficiency of NEPL distribution estimate. Therefore, the results of the simulation carried out for this study reveal that the process of estimating with reasonable degree of accuracy is feasible only when the size of the sample is comparatively large. The existence of the premature infant staying time data has been utilised to illustrate the specific manner in which the elaborated NEPL distribution has been suggested for being applied. Based on the discussions of the previous sections, it can be deduced that the NEPL distribution is also general in terms of its applications because it can deal with all forms of data that is, it does not distinguish between certainty, probabilities of uncertainties, ambiguties or imprecisions.

keywords: Exponentiated power Lomax distribution; neutrosophic statistics; survival analysis; premature infant.

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1 Introduction

Neutrosophy, philosophical and mathematical formation, was created by Florentin Smarandache (Smarandache, 1999). As for the general themes, it refers to the interaction of opposites on the one hand and the issue of the study of indeterminacy on the other hand. Consequently, there appears a new set of approaches, neoclassical logic and set theory, which are the extensions of classical logic and set theory and aim at solving the problems of inconsistency, indeterminacy, and imperfect information.

The basis of neutrosophic statistics is that often in the data, there is information, which is vague and cannot be quantified in the classic sense and therefore cannot be properly processed in the framework of traditional statistical methods. Neutrosophic statistics is used as a way to handle and do more comprehensive analysis on such data. The use of fuzzy logic was expanded by (Smarandache, 1999). to create neutrosophy, which enables the depiction of uncertainty, ambiguity, and contradiction.

Traditional analysis often suggests that the deeper the data, the clearer it is, hence very often each of the observations gets a numerical value. However, as it has been observed, in most real life settings, information can be ambiguous or about which there is limited detail given. To get around these constraints, neosophic statistics offers ways of dealing with the unpredictable, scarce, and contradictory data (Smarandache, 1999, 2014, 2022; Guan et al., 2019).

As already mentioned, neosophic statistics consider three measures that in some ways reflect the particularities of the evaluated propositions: truth membership, indeterminacy membership, and falsity membership. They all depict the extent of truth, openness, or falsehood that is correlated with a hypothesis or an observation. These degrees are represented in a manner similar to a fuzzy set by the membership functions [2, 3].

Neutrophic statistics are used in many different fields, including image processing, data mining, pattern recognition, and decision-making (Guan et al., 2019; Mao et al., 2020; Aslam, 2019; Taş et al., 2018). It provides a flexible mathematical tool for the analysis and modeling of complex systems with a high level of imprecision and uncertainty.

The survival statistics are among the essential aspects of neutrosophic information that have to be examined. Basically, the idea of survival analysis, often termed as event-time analysis or time-to-event analysis, deals with the assessment of time to certain event of interest. It is commonly applied in social science, engineering, medical research and other fields where the time related results are issued. Most of the time when conducting research where the temporal order is not certain or where subjects may not have the same subsequent follow-ups survival analysis proves to be of great use. It could also be the case that occurrence of a particular event of interest; an event say of failure, a relapse or even a death, or any other event of interest (Bibani et al., 2023).

There are many statistical distributions that are widely used in survival analysis to work on time-to-event data. This means that the features of the data and the assumptions made concerning the underlying survival process drives the choice of the distribution. These distributions are used for the assessment of time-to-event data in engineering, social sciences, and other medical disciplines (Ahmed et al., 2024b,a). Different distributions may be chosen depending on the given characteristics of the data by the researchers as well as the hypotheses appropriate to the study. The literature review reveals that many articles address neutrosophic probability distribution (Albassam et al., 2023; Alsoboh et al., 2023; Shah et al., 2022). Recently, neutrosophic distributions, such as the Neutrosophic Topp-Leone Extended Exponential distribution (Hammood et al., 2025), neutrosophic inverse Gompertz distribution (Al-Saqal et al., 2025), and Neutrosophic Beta-Lindley distribution (Algamal et al., 2024) have been proposed to model survival data.

Applications for the exponentiated power Lomax distribution can be found in many domains, including survival analysis. In this work, we extended the applications of the exponentiated power Lomax distribution to include neutrosophical data in interval form with a degree of indeterminacy. Many qualities are investigated under the newly proposed distribution and their applications are described with the help of simulated and real data application.

2 Neutrosophic exponentiated power Lomax distribution

Probability distributions help in the portrayal of uncertainty that is prevalent in the data set through depiction of the patterns of variation. In this regard distribution summaries, the observations into a mathematical form which contains a few unknown parameters and is the best possible understanding of the basic data generating mechanism. Survival time distribution which is the probability description of the behavior of length of life is to a certain extent depends on mode of succusses of the event under consideration. NEPL would likely combine the flexibility of neutrosophic logic with the adaptability of the exponentiated power Lomax distribution. This could provide a robust tool for analyzing complex, uncertain data in fields like economics, finance, or engineering, where both indeterminacy and heavy-tailed distributions are common.

From the given data set, the selection of the right distribution depends with the extent of prior information regarding the physical characteristics of the process underlying the observed data (Eliwa et al., 2021; Ijaz et al., 2020; Kharazmi et al., 2023).

The Lomax distribution (Pareto Type II) is a continuous probabilistic model used in modeling for heavy-tailed distributions (Alnssyan, 2023; Hamad, 2023; Abiodun and Ishaq, 2022). The Power Lomax distribution is an extension of the Lomax distribution which is commonly applied in actuarial sciences, business, and reliability engineering fields. Another advantage of this distribution is the introduction of another shape parameter which makes it possible to model lifetime data in the best manner as compared to the Lomax distribution (Rady et al., 2016). The four-parameter Lomax distribution, named as exponentiated power Lomax distribution (EPL), which was proposed by El-Monsef, Sweilam and Sabry (El-Monsef et al., 2021), is one of the survival time distributions. Figure 1 shows the pdf of the exponentiated power Lomax distribution

The concept of neutrosophic probability as a function $NP :\to [0,1]^3$ was originally presented by [2], where $V$is a neutrosophic sample space and defined the probability mapping to take the form $NP(\Omega) = (ch(\Omega), ch(neut\Omega), ch(anti\Omega)) = (\psi_1, \psi_2, \psi_3)$ with$0 \leq \psi_1, \psi_2, \psi_3 \leq 1$ and $0 \leq \psi_1 + \psi_2 + \psi_3 \leq 3$. The term Θ represents the set of

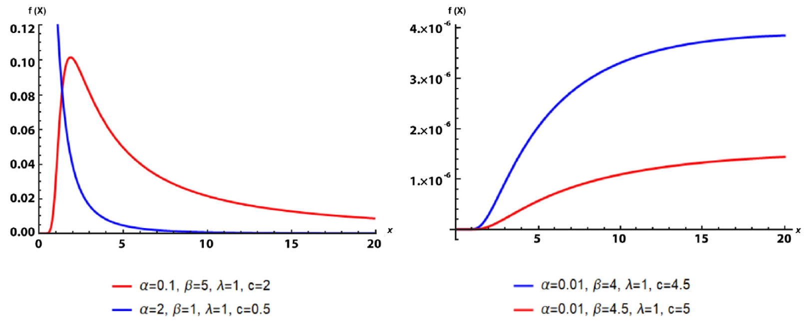

Figure 1: The pdf of exponentiated power Lomax distribution (El-Monsef et al., 2021)

sample space, R represents the set of real numbers, and ξ denotes a sample space event, X_N and Y_N denote neutrosophic random variable Hammood et al. (2025); Al-Saqal et al. (2025); Algamal et al. (2024).

Definition 1 Consider $X$is the real-valued crisp random variable, which has the following definition: $X : \Theta \rightarrow R$

where Θ is the event space and X_N neutrosophic random variable as follows:

$X_N : \Theta \rightarrow R\left(I\right)$ and $X_N = X + I$, where I represents indeterminacy.

Theorem 1 Let $X_N = X + I$ be the neutrosophic random variable and the CDF and pdf of X_N are (Granados, 2022), respectively

$F_{X_N}\left(x\right) = F_X\left(x - I\right)$, and $f_{X_N}\left(x\right) = f_X\left(x - I\right)$,

Theorem 2 Let $X_N = X + I$ be the neutrosophic random variable, then the expected value and variance can be derived as follows: $E\left(X_N\right) = E\left(X\right) + I$and $V\left(X_N\right) = V\left(X\right)$ [20].

By supposing the neutrosophic variable could be expressed as: $x_N = x_L + x_U I_N$where $I_N \in \{I_L, I_U\}$ and x_L and $x_U I_N$ denote the determined and indeterminate parts, respectively, the neutrosophic random variable $x_N \in \{x_L, x_U\}$ which follows the EPL distribution has neutrosophic parameters.

Let X be neutrosophic continuous random variable follows a neutrosophic EPL (NEPL) distribution, then its neutrosophic CDF and neutrosophic pdf are given by Eq.(1) and Eq. (1), respectively

$$F\left(x_N\right) = (1 - {\theta_N}^{\delta_N}\left(\theta_N + {x_N}^{\phi_N}\right)^{-\delta_N})^{\tau_N} \quad , \quad x_N > 0; \quad \delta_N, \phi_N, \theta_N, \tau_N > 0. \tag{1}$$

$$f\left(x_N\right) = \delta_N \phi_N \tau_N {\theta_N}^{\delta_N} {x_N}^{\phi_N - 1}\left(\theta_N + {x_N}^{\phi_N}\right)^{-\delta_N - 1}(1 - {\theta_N}^{\delta_N}\left(\theta_N + {x_N}^{\phi_N}\right)^{-\delta_N})^{\tau_N} \tag{2}$$

The NEPL distribution provides better capability to model complex datasets than traditional distribution models do. NEPL distribution extends traditional models thus providing adequate capabilities for capturing various data patterns found in practical applications. Figure 1 shows the NEPL distribution for different values of its parameters.

Relating to Eq.(1) and Eq.(2), the neutrosophic survival and hazard functions of the NEPL distribution are defined in Eq.(3) and Eq.(4), respectively,

$$S(x_N) = 1 - \left(1 - {\theta_N}^{\delta_N}\left(\theta_N + {x_N}^{\phi_N}\right)^{-\delta_N}\right)^{\tau_N}, \tag{3}$$

$$h(x_N) = \frac{\delta_N \phi_N \tau_N {\theta_N}^{\delta_N} {x_N}^{\phi_N - 1}\left(\theta_N + {x_N}^{\phi_N}\right)^{-\delta_N - 1}(1 - {\theta_N}^{\delta_N}\left(\theta_N + {x_N}^{\phi_N}\right)^{-\delta_N})^{\tau_N - 1}}{1 - (1 - {\theta_N}^{\delta_N}(\theta_N + {x_N}^{\phi_N})^{-\delta_N})^{\tau_N}} \tag{4}$$

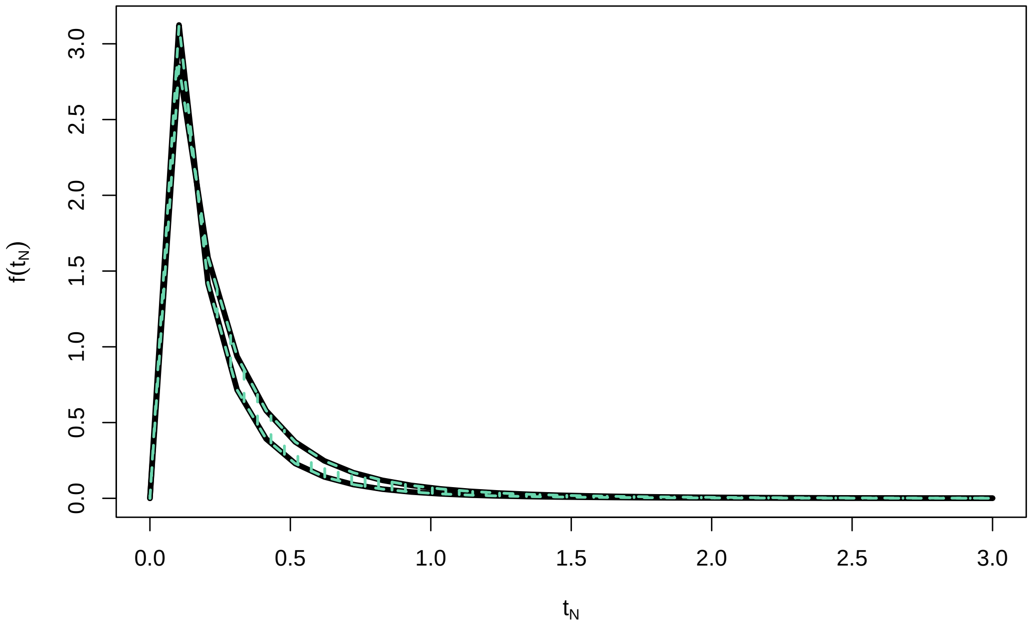

Figure 2: The pdf of NEPL when $\delta_N \in [0.15, 0.21]$, $\phi_N \in [5.1, 5.3]$, $\theta_N \in [1, 1.3]$, and $\tau_N \in [2, 2.4]$

3 Parameter Estimation of NEPL Distribution

Five methods for estimating the NEPL distribution parameters are described:(1) the maximum likelihood Method (MLE), (2) Anderson Darling method (AD), (3) Cramér-von Mises method (CVM), (4) weighted least-squares method (WLS), and (5) the maximum product spacing method (MPS).

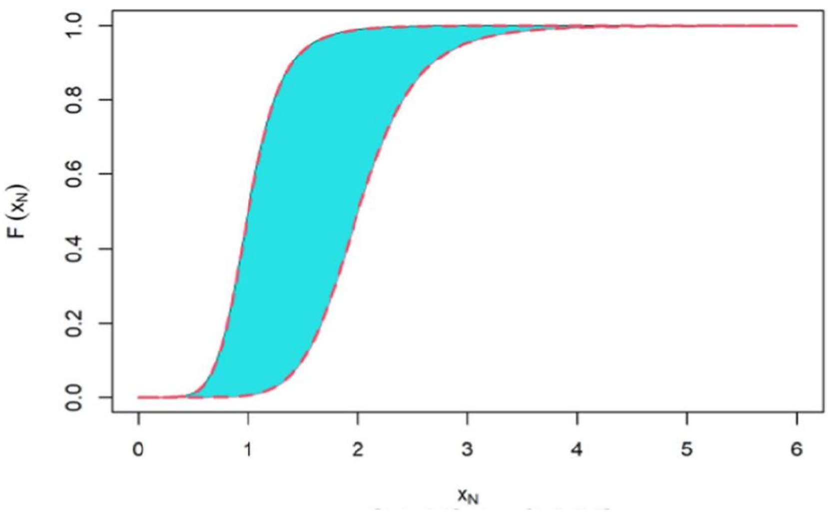

Figure 3: The CDF of f NEPL when $\delta_N \in [0.15, 0.21]$, $\phi_N \in [5.1, 5.3]$, $\theta_N \in [1, 1.3]$, and $\tau_N \in [2, 2.4]$

3.1 MLE method

Assuming each of the random samples $x_1, x_2, \ldots, x_n$ follows NEPL distribution, the log-likelihood function is given by

$$\begin{aligned}\ln L\left(\delta_N, \phi_N, \theta_N, \tau_N\right) = n_N \log\left(\delta_N \phi_N \tau_N \theta_N^{\delta_N}\right) \\ + \left(\tau_N - 1\right) \sum_{i_N=1}^{n_N} \log\left(1 - \theta_N^{\delta_N}\left(x_{i_N}^{\phi_N} + \theta_N\right)^{-\delta_N}\right) \\ - \left(\delta_N + 1\right) \sum_{i_N=1}^{n_N} \log\left(x_{i_N}^{\phi_N} + \theta_N\right) + \left(\phi_N + 1\right) \sum_{i_N=1}^{n_N} \log\left(x_{i_N}\right). \quad (5)\end{aligned}$$

The MLE of the parameters $\hat{\delta}_N$, $\hat{\theta}_N$, $\hat{\tau}_N$, and $\hat{\phi}_N$ are the solutions of the following simultaneous equations:

$$\frac{\partial \ln L(\delta_N, \phi_N, \theta_N, \tau_N)}{\partial \delta_N} = 0, \ \frac{\partial \ln L(\delta_N, \phi_N, \theta_N, \tau_N)}{\partial \theta_N} = 0,$$
$$\frac{\partial \ln L(\delta_N, \phi_N, \theta_N, \tau_N)}{\partial \phi_N} = 0, \ \frac{\partial \ln L(\delta_N, \phi_N, \theta_N, \tau_N)}{\partial \tau_N} = 0$$

$$\frac{\partial \ln L\left(\delta_N,\phi_N,\theta_N,\tau_N\right)}{\partial \delta_N} = \frac{n_N}{\delta_N} + n_N \log\left(\theta_N\right) - \sum_{i_N=1}^{n_N} \log\left(x_{i_N}^{\phi_N} + \theta_N\right)$$
$$+ \left(\tau_N - 1\right) \sum_{i_N=1}^{n_N} \frac{\theta_N^{\delta_N}\left(x_{i_N}^{\phi_N}+\theta_N\right)^{-\delta_N} \log\left(x_{i_N}^{\phi_N}+\theta_N\right) - \theta_N^{\delta_N}\log\left(\theta_N\right)\left(x_{i_N}^{\phi_N}+\theta_N\right)^{-\delta_N}}{1-\theta_N^{\delta_N}\left(x_{i_N}^{\phi_N}+\theta_N\right)^{-\delta_N}} \tag{6}$$

$$\frac{\partial \ln L(\delta_N,\phi_N,\theta_N,\tau_N)}{\partial \theta_N} = \frac{n_N}{\phi_N} + \sum_{i_N=1}^{n_N} \log\left(x_{i_N}\right) - \left(\delta_N+1\right)\sum_{i_N=1}^{n_N} \frac{x_{i_N}^{\phi_N}\log\left(x_{i_N}\right)}{x_{i_N}^{\phi_N}+\theta_N} + \left(\tau_N-1\right)\sum_{i_N=1}^{n_N} \frac{\delta_N {\theta_N}^{\delta_N} x_{i_N}^{\phi_N}\log\left(x_{i_N}\right)\left(x_{i_N}^{\phi_N}+\theta_N\right)^{-\delta_N-1}}{1-{\theta_N}^{\delta_N}\left(x_i^{\phi}+\theta_N\right)^{-\delta_N}} \tag{7}$$

$$\frac{\partial \ln L(\delta_N,\phi_N,\theta_N,\tau_N)}{\partial \phi_N} = \frac{\delta_N n_N}{\phi_N} - \left(\delta_N+1\right)\sum_{i_N=1}^{n_N} \frac{1}{x_{i_N}^{\phi_N}+\theta_N} + \left(\tau_N-1\right) \sum_{i_N=1}^{n_N} \frac{\delta_N {\theta_N}^{\delta_N}\left(x_{i_N}^{\phi_N}+\theta_N\right)^{-\delta_N-1} - \delta_N {\theta_N}^{\delta_N-1}\left(x_{i_N}^{\phi_N}+\theta_N\right)^{-\delta_N}}{1-{\theta_N}^{\delta_N}\left(x_{i_N}^{\phi_N}+\theta_N\right)^{-\delta_N}} \tag{8}$$

$$\frac{\partial \ln L\left(\delta_N,\phi_N,\theta_N,\tau_N\right)}{\partial \tau_N} = \frac{n_N}{\tau_N} + \sum_{i_N=1}^{n_N} \log\left(1-{\theta_N}^{\delta_N}\left(x_{i_N}^{\phi_N}+\theta_N\right)^{-\delta_N}\right) \tag{9}$$

3.2 AD method

The AD estimates of the parameters $\hat{\delta}_N$, $\hat{\theta}_N$, $\hat{\tau}_N$, and $\hat{\phi}_N$ are attained by minimizing the following equation with respect to the unknown parameters:

$$AD\left(\delta_N,\phi_N,\theta_N,\tau_N\right) = -n - \sum_{i_N=1}^{n} \frac{\left(2i_N-1\right)}{n} \left[\begin{array}{c} \log\left(F\left(X_{i:nN};\delta_N,\phi_N,\theta_N,\tau_N\right)\right) \\ +\log\left(1-F\left(X_{i:nN};\delta_N,\phi_N,\theta_N,\tau_N\right)\right) \end{array}\right] \tag{10}$$

3.3 CVM method

The CVM estimates $\hat{\delta}_N$, $\hat{\theta}_N$, $\hat{\tau}_N$, and $\hat{\phi}_N$ are derived by minimizing the following expression with respect to NEPL parameters as:

$$CVM\left(\delta_N,\phi_N,\theta_N,\tau_N\right) = \frac{1}{12n} + \sum_{i_N=1}^{n}\left(F\left(X_{i:nN};\delta_N,\phi_N,\theta_N,\tau_N\right) - \frac{2i_N-1}{2n}\right)^2 \tag{11}$$

3.4 WLS method

The WLS estimates $\hat{\delta}_N$, $\hat{\theta}_N$, $\hat{\tau}_N$, and $\hat{\phi}_N$ are derived by minimizing Eq. (12) with respect to NEPL parameters as:

$$WLS\left(\delta_N,\phi_N,\theta_N,\tau_N\right) = \sum_{i_N=1}^{n} \frac{\left(1+n\right)^2\left(2+n\right)}{i_N\left(n-i_N+1\right)}\left[F\left(X_{i:nN};\delta_N,\phi_N,\theta_N,\tau_N\right) - \frac{i_N}{n+1}\right]^2. \tag{12}$$

3.5 MPS method

The estimation of NEPL parameters using MPS method can be obtained by maximizing Eq.(13)as:

$$MPS\left(\delta_N,\phi_N,\theta_N,\tau_N\right)=\frac{1}{n+1}\sum_{i_N=1}^{n+1}\log\Big(F\left(X_{i:nN};\delta_N,\phi_N,\theta_N,\tau_N\right) - F\left(X_{i-1:nN};\delta_N,\phi_N,\theta_N,\tau_N\right)\Big), \tag{13}$$

4 Simulation results

To investigate the efficacy of the NEPL of the neutrosophic parameters δ_N, θ_N, τ_N, and ϕ_N of the suggested EPL, simulation research is conducted in this section. A random sample of sizes, n = 30, 50, 150 and 250, is created from NEPL using different amalgams of neutrosophic parameters for the simulation. Estimated MLEs, AD, CVM, WLS, and MPS of the neutrosophic parameters for 1000 replications at different sample sizes using simulated data. Thus, for all sample sizes, the neutrosophic mean square error (NMSE) and the neutrosophic average bias (NAB) are derived. The superior neutrosophic estimator's properties are evaluated using the estimations of NAB and NMSE (Mustafa et al., 2023; Alanaz and Algamal, 2023; Alanaz et al., 2023).

Three cases of the NEPL neutrosophic parameters are determined: Case (1): $\delta_N \in [0.11, 0.17]$, $\phi_N \in [5, 5.4]$, $\theta_N \in [1, 1.5]$, and $\tau_N \in [2, 2.5]$. Case (2): $\delta_N \in [0.15, 0.20]$, $\phi_N \in [5.4, 6]$, $\theta_N \in [1.5, 2]$, and $\tau_N \in [2.3, 2.8]$. And Case (3): $\delta_N \in [0.20, 0.25]$, $\phi_N \in [5.7, 6.3]$, $\theta_N \in [1.8, 2.4]$, and $\tau_N \in [2.6, 3]$. The results are given in Tables 1 – 6.

From Tables 1, 3, and 5, in terms of NAB, it is seen that, as predicted, the NAB for $\hat{\delta}_N$, $\hat{\theta}_N$, $\hat{\tau}_N$, and $\hat{\phi}_N$decrease as sample sizes rise. It can also be deduced from Tables 1, 3, and 5 that the NAB values of the five estimators are varying as expected. The NAB values of $\hat{\delta}_N$, $\hat{\theta}_N$, $\hat{\tau}_N$, and $\hat{\phi}_N$for MPS, WLS, and CVM are higher than MLE and AD estimators.

Concerning the NMSE values, and for all sample sizes, the MLE estimator of $\hat{\delta}_N$, $\hat{\theta}_N$, $\hat{\tau}_N$, and $\hat{\phi}_N$has the smallest values comparing with AD, CVM, WLS, and MPS estimators. Further, it is noticed from Tables 2, 4, and 6 that when the$\hat{\delta}_N$, $\hat{\theta}_N$, $\hat{\tau}_N$, and $\hat{\phi}_N$increase regardless the values of the n, the NMSE are decreasing.

5 Real Application

From our study, we have used premature infant staying time data that we gathered from Mosul hospital, Iraq for about four months to apply our proposed NEPL distribution. The time corresponds to the number of days that the premature infant is alive after discharge from the hospital. The subject population in the study is 100 premature infants. However, premature infant times are not recorded accurately, the member countries need to develop clear and accurate time definitions. Therefore, defining the

Table 1: Average values of NAB for case 1

n		30	50	150	250
MLE	$\hat{\delta}_N$	[0.1082,0.1097]	[0.1058,0.1073]	[0.1034,0.1049]	[0.101,0.1025]
	$\hat{\phi}_N$	[0.1183,0.1209]	[0.1159,0.1185]	[0.1135,0.1161]	[0.1111,0.1137]
	$\hat{\theta}_N$	[0.1103,0.1118]	[0.1079,0.1094]	[0.1055,0.107]	[0.1031,0.1046]
	$\hat{\tau}_N$	[0.1204,0.123]	[0.118,0.1206]	[0.1156,0.1182]	[0.1132,0.1158]
AD	$\hat{\delta}_N$	[0.1793,0.1808]	[0.1769,0.1784]	[0.1745,0.176]	[0.1721,0.1736]
	$\hat{\phi}_N$	[0.1894,0.192]	[0.187,0.1896]	[0.1846,0.1872]	[0.1822,0.1848]
	$\hat{\theta}_N$	[0.1814,0.1829]	[0.179,0.1805]	[0.1766,0.1781]	[0.1742,0.1757]
	$\hat{\tau}_N$	[0.1915,0.1941]	[0.1891,0.1917]	[0.1867,0.1893]	[0.1843,0.1869]
CVM	$\hat{\delta}_N$	[0.2504,0.2519]	[0.248,0.2495]	[0.2456,0.2471]	[0.2432,0.2447]
	$\hat{\phi}_N$	[0.2605,0.2631]	[0.2581,0.2607]	[0.2557,0.2583]	[0.2533,0.2468]
	$\hat{\theta}_N$	[0.2525,0.254]	[0.2501,0.2516]	[0.2477,0.2492]	[0.2453,0.258]
	$\hat{\tau}_N$	[0.2626,0.2652]	[0.2602,0.2628]	[0.2578,0.2604]	[0.2554,0.3158]
WLS	$\hat{\delta}_N$	[0.3215,0.323]	[0.3191,0.3206]	[0.3167,0.3182]	[0.3143,0.327]
	$\hat{\phi}_N$	[0.3316,0.3342]	[0.3292,0.3318]	[0.3268,0.3294]	[0.3244,0.3179]
	$\hat{\theta}_N$	[0.3236,0.3251]	[0.3212,0.3227]	[0.3188,0.3203]	[0.3164,0.3291]
	$\hat{\tau}_N$	[0.3337,0.3363]	[0.3313,0.3339]	[0.3289,0.3315]	[0.3265,0.3869]
MPS	$\hat{\delta}_N$	[0.3926,0.3941]	[0.3902,0.3917]	[0.3878,0.3893]	[0.3854,0.3981]
	$\hat{\phi}_N$	[0.4027,0.4053]	[0.4003,0.4029]	[0.3979,0.4005]	[0.3955,0.389]
	$\hat{\theta}_N$	[0.3947,0.3962]	[0.3923,0.3938]	[0.3899,0.3914]	[0.3875,0.4002]
	$\hat{\tau}_N$	[0.4048,0.4074]	[0.4024,0.405]	[0.4,0.4026]	[0.3976,0.2559]

Table 2: Average values of NMSE for case 1

n		30	50	150	250
MLE	$\hat{\delta}_N$	[0.2416,0.2431]	[0.2392,0.2407]	[0.2368,0.2383]	[0.2344,0.2359]
	$\hat{\phi}_N$	[0.2517,0.2543]	[0.2493,0.2519]	[0.2469,0.2495]	[0.2445,0.2471]
	$\hat{\theta}_N$	[0.2437,0.2452]	[0.2413,0.2428]	[0.2389,0.2404]	[0.2365,0.238]
	$\hat{\tau}_N$	[0.2538,0.2564]	[0.2514,0.254]	[0.249,0.2516]	[0.2466,0.2492]
AD	$\hat{\delta}_N$	[0.3127,0.3142]	[0.3103,0.3118]	[0.3079,0.3094]	[0.3055,0.307]
	$\hat{\phi}_N$	[0.3228,0.3254]	[0.3204,0.323]	[0.318,0.3206]	[0.3156,0.3182]
	$\hat{\theta}_N$	[0.3148,0.3163]	[0.3124,0.3139]	[0.31,0.3115]	[0.3076,0.3091]
	$\hat{\tau}_N$	[0.3249,0.3275]	[0.3225,0.3251]	[0.3201,0.3227]	[0.3177,0.3203]
CVM	$\hat{\delta}_N$	[0.3838,0.3853]	[0.3814,0.3829]	[0.379,0.3805]	[0.3766,0.3781]
	$\hat{\phi}_N$	[0.3939,0.3965]	[0.3915,0.3941]	[0.3891,0.3917]	[0.3867,0.3893]
	$\hat{\theta}_N$	[0.2859,0.3874]	[0.3835,0.385]	[0.3811,0.3826]	[0.3787,0.3802]
	$\hat{\tau}_N$	[0.396,0.3986]	[0.3936,0.3962]	[0.3912,0.3938]	[0.3888,0.3914]
WLS	$\hat{\delta}_N$	[0.4549,0.4564]	[0.4525,0.454]	[0.4501,0.4516]	[0.4477,0.4492]
	$\hat{\phi}_N$	[0.465,0.4676]	[0.4626,0.4652]	[0.4602,0.4628]	[0.4578,0.4604]
	$\hat{\theta}_N$	[0.457,0.4585]	[0.4546,0.4561]	[0.2522,0.4537]	[0.4498,0.4513]
	$\hat{\tau}_N$	[0.4671,0.4697]	[0.4647,0.4673]	[0.4623,0.4649]	[0.4599,0.4625]
MPS	$\hat{\delta}_N$	[0.256,0.5275]	[0.5236,0.5251]	[0.5212,0.5227]	[0.5188,0.5203]
	$\hat{\phi}_N$	[0.5361,0.5387]	[0.5337,0.5363]	[0.5313,0.5339]	[0.5289,0.5315]
	$\hat{\theta}_N$	[0.5281,0.5296]	[0.5257,0.5272]	[0.5233,0.5248]	[0.5209,0.5224]
	$\hat{\tau}_N$	[0.5382,0.5408]	[0.5358,0.5384]	[0.5334,0.536]	[0.531,0.5336]

Table 3: Average values of NAB for case 2

n		30	50	150	250
MLE	$\hat{\delta}_N$	[0.1035,0.105]	[0.1011,0.1026]	[0.0987,0.1002]	[0.0963,0.0978]
	$\hat{\phi}_N$	[0.1136,0.1162]	[0.1112,0.1138]	[0.1088,0.1114]	[0.1064,0.109]
	$\hat{\theta}_N$	[0.1056,0.1071]	[0.1032,0.1047]	[0.1008,0.1023]	[0.0984,0.0999]
	$\hat{\tau}_N$	[0.1157,0.1183]	[0.1133,0.1159]	[0.1109,0.1135]	[0.1085,0.1111]
AD	$\hat{\delta}_N$	[0.1746,0.1761]	[0.1722,0.1737]	[0.1698,0.1713]	[0.1674,0.1689]
	$\hat{\phi}_N$	[0.1847,0.1873]	[0.1823,0.1849]	[0.1799,0.1825]	[0.1775,0.1801]
	$\hat{\theta}_N$	[0.1767,0.1782]	[0.1743,0.1758]	[0.1719,0.1734]	[0.1695,0.171]
	$\hat{\tau}_N$	[0.1868,0.1894]	[0.1844,0.187]	[0.182,0.1846]	[0.1796,0.1822]
CVM	$\hat{\delta}_N$	[0.2457,0.2472]	[0.2433,0.2448]	[0.2409,0.2424]	[0.2385,0.24]
	$\hat{\phi}_N$	[0.2558,0.2584]	[0.2534,0.256]	[0.251,0.2536]	[0.2486,0.2512]
	$\hat{\theta}_N$	[0.2478,0.2493]	[0.2454,0.2469]	[0.243,0.2445]	[0.2406,0.2421]
	$\hat{\tau}_N$	[0.2579,0.2605]	[0.2555,0.2581]	[0.2531,0.2557]	[0.2507,0.2533]
WLS	$\hat{\delta}_N$	*[0.3168,0.3183]*	*[0.3144,0.3159]*	*[0.312,0.3135]*	*[0.3096,0.3111]*
	$\hat{\phi}_N$	*[0.3269,0.3295]*	*[0.3245,0.3271]*	*[0.3221,0.3247]*	*[0.3197,0.3223]*
	$\hat{\theta}_N$	*[0.3189,0.3204]*	*[0.3165,0.318]*	*[0.3141,0.3156]*	*[0.3117,0.3132]*
	$\hat{\tau}_N$	*[0.329,0.3316]*	*[0.3266,0.3292]*	*[0.3242,0.3268]*	*[0.3218,0.3244]*
MPS	$\hat{\delta}_N$	[0.3879,0.3894]	[0.3855,0.387]	[0.3831,0.3846]	[0.3807,0.3822]
	$\hat{\phi}_N$	[0.398,0.4006]	[0.3956,0.3982]	[0.3932,0.3958]	[0.3908,0.3934]
	$\hat{\theta}_N$	[0.39,0.3915]	[0.3876,0.3891]	[0.3852,0.3867]	[0.3828,0.3843]
	$\hat{\tau}_N$	[0.4001,0.4027]	[0.3977,0.4003]	[0.3953,0.3979]	[0.3929,0.3955]

Table 4: Average values of NMSE for case 2

n		30	50	150	250
MLE	$\hat{\delta}_N$	[0.2354,0.2369]	[0.233,0.2345]	[0.2306,0.2321]	[0.2282,0.2297]
	$\hat{\phi}_N$	[0.2455,0.2481]	[0.2431,0.2457]	[0.2407,0.2433]	[0.2383,0.2409]
	$\hat{\theta}_N$	[0.2375,0.239]	[0.2351,0.2366]	[0.2327,0.2342]	[0.2303,0.2318]
	$\hat{\tau}_N$	[0.2476,0.2502]	[0.2452,0.2478]	[0.2428,0.2454]	[0.2404,0.243]
AD	$\hat{\delta}_N$	[0.3065,0.308]	[0.3041,0.3056]	[0.3017,0.3032]	[0.2993,0.3008]
	$\hat{\phi}_N$	[0.3166,0.3192]	[0.3142,0.3168]	[0.3118,0.3144]	[0.3094,0.312]
	$\hat{\theta}_N$	[0.3086,0.3101]	[0.3062,0.3077]	[0.3038,0.3053]	[0.3014,0.3029]
	$\hat{\tau}_N$	[0.3187,0.3213]	[0.3163,0.3189]	[0.3139,0.3165]	[0.3115,0.3141]
CVM	$\hat{\delta}_N$	[0.3776,0.3791]	[0.3752,0.3767]	[0.3728,0.3743]	[0.3704,0.3719]
ne	$\hat{\phi}_N$	[0.3877,0.3903]	[0.3853,0.3879]	[0.3829,0.3855]	[0.3805,0.3831]
	$\hat{\theta}_N$	[0.3797,0.3812]	[0.3773,0.3788]	[0.3749,0.3764]	[0.3725,0.374]
	$\hat{\tau}_N$	[0.3898,0.3924]	[0.3874,0.39]	[0.385,0.3876]	[0.3826,0.3852]
WLS	$\hat{\delta}_N$	[0.4487,0.4502]	[0.4463,0.4478]	[0.4439,0.4454]	[0.4415,0.443]
	$\hat{\phi}_N$	[0.4588,0.4614]	[0.4564,0.459]	[0.454,0.4566]	[0.4516,0.4542]
e	$\hat{\theta}_N$	[0.4508,0.4523]	[0.4484,0.4499]	[0.446,0.4475]	[0.4436,0.4451]
	$\hat{\tau}_N$	[0.4609,0.4635]	[0.4585,0.4611]	[0.4561,0.4587]	[0.4537,0.4563]
MPS	$\hat{\delta}_N$	[0.5198,0.5213]	[0.5174,0.5189]	[0.515,0.5165]	[0.5126,0.5141]
	$\hat{\phi}_N$	[0.5299,0.5325]	[0.5275,0.5301]	[0.5251,0.5277]	[0.5227,0.5253]
	$\hat{\theta}_N$	[0.5219,0.5234]	[0.5195,0.521]	[0.5171,0.5186]	[0.5147,0.5162]
	$\hat{\tau}_N$	[0.532,0.5346]	[0.5296,0.5322]	[0.5272,0.5298]	[0.5248,0.5274]

Table 5: Average values of NAB for case 3

n		30	50	150	250
MLE	$\hat{\delta}_N$	[0.0933,0.0948]	[0.0909,0.0924]	[0.0885,0.09]	[0.0861,0.0876]
	$\hat{\phi}_N$	[0.1034,0.106]	[0.101,0.1036]	[0.0986,0.1012]	[0.0962,0.0988]
	$\hat{\theta}_N$	[0.0954,0.0969]	[0.093,0.0945]	[0.0906,0.0921]	[0.0882,0.0897]
	$\hat{\tau}_N$	[0.1055,0.1081]	[0.1031,0.1057]	[0.1007,0.1033]	[0.0983,0.1009]
AD	$\hat{\delta}_N$	[0.1644,0.1659]	[0.162,0.1635]	[0.1596,0.1611]	[0.1572,0.1587]
	$\hat{\phi}_N$	[0.1745,0.1771]	[0.1721,0.1747]	[0.1697,0.1723]	[0.1673,0.1699]
	$\hat{\theta}_N$	[0.1665,0.168]	[0.1641,0.1656]	[0.1617,0.1632]	[0.1593,0.1608]
	$\hat{\tau}_N$	[0.1766,0.1792]	[0.1742,0.1768]	[0.1718,0.1744]	[0.1694,0.172]
CVM	$\hat{\delta}_N$	[0.2355,0.237]	[0.2331,0.2346]	[0.2307,0.2322]	[0.2283,0.2298]
	$\hat{\phi}_N$	[0.2456,0.2482]	[0.2432,0.2458]	[0.2408,0.2434]	[0.2384,0.241]
	$\hat{\theta}_N$	[0.2376,0.2391]	[0.2352,0.2367]	[0.2328,0.2343]	[0.2304,0.2319]
	$\hat{\tau}_N$	[0.2477,0.2503]	[0.2453,0.2479]	[0.2429,0.2455]	[0.2405,0.2431]
WLS	$\hat{\delta}_N$	[0.3066,0.3081]	[0.3042,0.3057]	[0.3018,0.3033]	[0.2994,0.3009]
	$\hat{\phi}_N$	[0.3167,0.3193]	[0.3143,0.3169]	[0.3119,0.3145]	[0.3095,0.3121]
	$\hat{\theta}_N$	[0.3087,0.3102]	[0.3063,0.3078]	[0.3039,0.3054]	[0.3015,0.303]
	$\hat{\tau}_N$	[0.3188,0.3214]	[0.3164,0.319]	[0.314,0.3166]	[0.3116,0.3142]
MPS	$\hat{\delta}_N$	[0.3777,0.3792]	[0.3753,0.3768]	[0.3729,0.3744]	[0.3705,0.372]
	$\hat{\phi}_N$	[0.3878,0.3904]	[0.3854,0.388]	[0.383,0.3856]	[0.3806,0.3832]
	$\hat{\theta}_N$	[0.3798,0.3813]	[0.3774,0.3789]	[0.375,0.3765]	[0.3726,0.3741]
	$\hat{\tau}_N$	[0.3899,0.3925]	[0.3875,0.3901]	[0.3851,0.3877]	[0.3827,0.3853]

Table 6: Average values of NMSE for case 3

n		30	50	150	250
MLE	$\hat{\delta}_N$	[0.2213,0.2228]	[0.2189,0.2204]	[0.2165,0.218]	[0.2141,0.2156]
	$\hat{\phi}_N$	[0.2314,0.234]	[0.229,0.2316]	[0.2266,0.2292]	[0.2242,0.2268]
	$\hat{\theta}_N$	[0.2234,0.2249]	[0.221,0.2225]	[0.2186,0.2201]	[0.2162,0.2177]
	$\hat{\tau}_N$	[0.2335,0.2361]	[0.2311,0.2337]	[0.2287,0.2313]	[0.2263,0.2289]
AD	$\hat{\delta}_N$	[0.2924,0.2939]	[0.29,0.2915]	[0.2876,0.2891]	[0.2852,0.2867]
	$\hat{\phi}_N$	[0.3025,0.3051]	[0.3001,0.3027]	[0.2977,0.3003]	[0.2953,0.2979]
	$\hat{\theta}_N$	[0.2945,0.296]	[0.2921,0.2936]	[0.2897,0.2912]	[0.2873,0.2888]
	$\hat{\tau}_N$	[0.3046,0.3072]	[0.3022,0.3048]	[0.2998,0.3024]	[0.2974,0.3]
CVM	$\hat{\delta}_N$	[0.3635,0.365]	[0.3611,0.3626]	[0.3587,0.3602]	[0.3563,0.3578]
	$\hat{\phi}_N$	[0.3736,0.3762]	[0.3712,0.3738]	[0.3688,0.3714]	[0.3664,0.369]
	$\hat{\theta}_N$	[0.3656,0.3671]	[0.3632,0.3647]	[0.3608,0.3623]	[0.2584,0.3599]
	$\hat{\tau}_N$	[0.3757,0.3783]	[0.3733,0.3759]	[0.3709,0.3735]	[0.3685,0.3711]
WLS	$\hat{\delta}_N$	[0.4346,0.4361]	[0.4322,0.4337]	[0.4298,0.4313]	[0.4274,0.4289]
	$\hat{\phi}_N$	[0.4447,0.4473]	[0.4423,0.4449]	[0.4399,0.4425]	[0.4975,0.4401]
	$\hat{\theta}_N$	[0.4367,0.4382]	[0.4343,0.4358]	[0.4319,0.4334]	[0.4295,0.431]
	$\hat{\tau}_N$	[0.4468,0.4494]	[0.444,0.447]	[0.442,0.4446]	[0.4396,0.4422]
MPS	$\hat{\delta}_N$	[0.5057,0.5072]	[0.5033,0.5048]	[0.5009,0.5024]	[0.4685,0.5]
	$\hat{\phi}_N$	[0.5158,0.5184]	[0.5134,0.516]	[0.511,0.5136]	[0.5086,0.5112]
	$\hat{\theta}_N$	[0.5078,0.5093]	[0.5054,0.5069]	[0.503,0.5045]	[0.5006,0.5021]
	$\hat{\tau}_N$	[0.5179,0.5205]	[0.5155,0.5181]	[0.5131,0.5157]	[0.5107,0.5133]

number of days from the birth of a premature infant with alive discharge without having explicit information is always problematic supervising unpredictable, insufficient, and incongruent data. The data related to exploratory factor analysis is given in Table 7.

Table 7: The staying time data

0.87,1.54, 1.45, [1.83,2.01], 1.15, 1.81, [1.47,1.62], 1.63, 1.09, [0.86,1.2], 1.00, 1.39, 1.06, 1.08, 0.81,1.19,0.88, 1.65, 0.84, 1.47, 1.59, 0.81, 1.00, 1.30, [1.47,1.70], 1.09, 1.80, 1.08, 1.11, [1.00,1.14],1.27, 1.23, 0.81,1.42, 0.94, 1.32,1.19, 1.30, 1.28, 1.00, 0.96, 1.21, 2.36, 1.04, 1.03, [0.81,1.1], 1.20, 1.26, 1.42, 1.87, 1.16,[0.85,1.2], [0.98,1.3], 1.63, 1.58, 1.57, 1.84, 1.32, 1.16, [0.93,1.4], 1.38, 0.85,1.14,1.80,1.57,[1.39,1.62],0.88,1.26,2.18,1.28,0.99,0.83,0.84,1.63,1.06, 1.84,1.22, 1.18, 1.85,.87,1.53,1.07,1.22, 1.03,0.85,1.47,[0.93,1.3],1.32, 1.12, [0.94,1.21], 2.30,1.13,0,81,1.80, 1.62,1.26,[1.01,1.24], 1.51, 1.87, 1.58

An informal graphical technique has been utilized to show that the EPL distribution is one of the plausible models for explaining the premature infant staying time data. Figure 2 displays a visual fit of the EPL distribution. Further, the χ^2 test for the goodness of fit shows that the premature infant staying time data follows EPL distribution with p-value=0.764. A descriptive assessment of the premature infant staying time data using NEPL is shown in Table 8. Table 8 makes it abundantly evident that uncertainties taken into account in the observed sample are the cause of discrepancies in a number of the critical numerical statistics of the failure times data. Further, it is more clearly shown from Table 8 that there are high varies among the estimation methods in estimating the NEPL distribution parameters $\hat{\delta}_N$, $\hat{\theta}_N$, $\hat{\tau}_N$, and $\hat{\phi}_N$.

In terms of survival probability, Figure 3 displays the survival curve for the five estimation methods. It can be observed that the neutrosophic survival curve using MLE methods shows higher probability than the others. This suggests that the neutrosophic MLE is better than the other four methods. Based on this observation, Figure 4 depicts the margin of the survival function between lower and upper the premature infant staying time data.

6 Conclusions

This paper presents an interesting extension known as the neutrosophic exponentiated power Lomax distribution. The concepts of neutrosophic calculus serve as the foundation for this new extension. The neutrosophic paradigm has been used to investigate a number of estimation methods. The study's numerical examples showed that NEPL distribution's theoretical conclusions are flexible and applicable to a wide range of data. The simulation study's findings suggest that a large sample size can yield accurate estimations. The premature infant staying time data have been employed to explicate the practical implementation of the suggested NEPL distribution. The application section has demonstrated that the NEPL distribution is capable of analyzing both classical

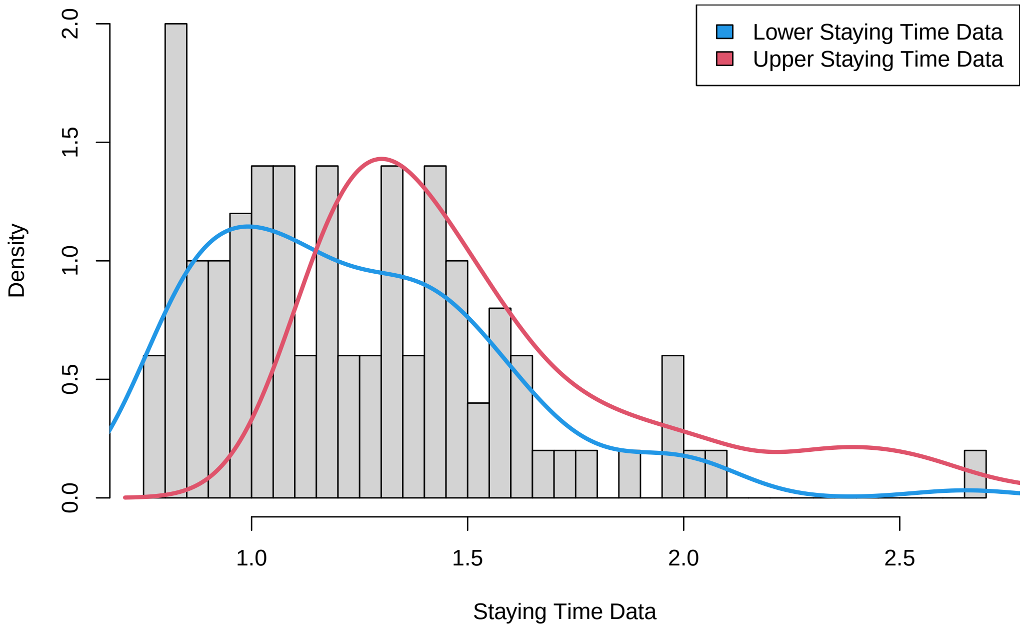

Figure 4: Fitting of EPL distribution of staying time data

Table 8: The estimated parameters of NEPL distribution

Method	Estimated values			
	$\hat{\delta}_N$	$\hat{\theta}_N$	$\hat{\phi}_N$	$\hat{\tau}_N$
MLE	[0.157,0.164]	[1.53,1.58]	[5.51,5.93]	[2.35,2.41]
AD	[0.162,0.169]	[1.55,1.61]	[5.82,6.34]	[2.37,2.44]
CVM	[0.171,0.178]	[1.58,1.67]	[6.52,6.67]	[2.42,2.51]
WLS	[0.176,0.182]	[1.62,1.69]	[6.66,6.71]	[2.55,2.62]
MPS	[0.180,0.188]	[1.66,1.74]	[6.73,6.78]	[2.58,2.67]

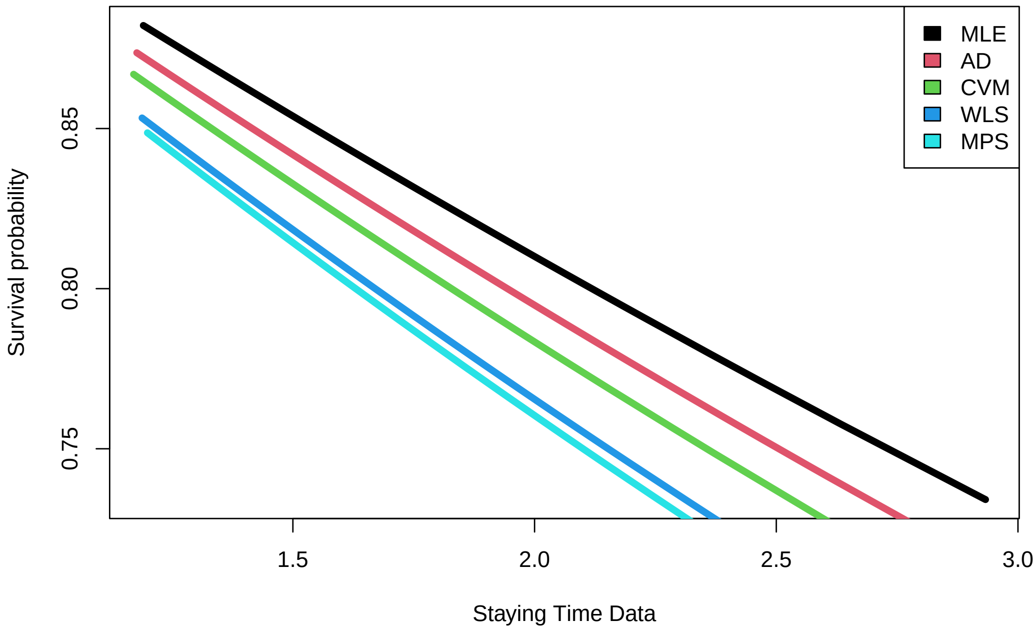

Figure 5: The survival curve plot for the NEPL distribution under several estimation methods

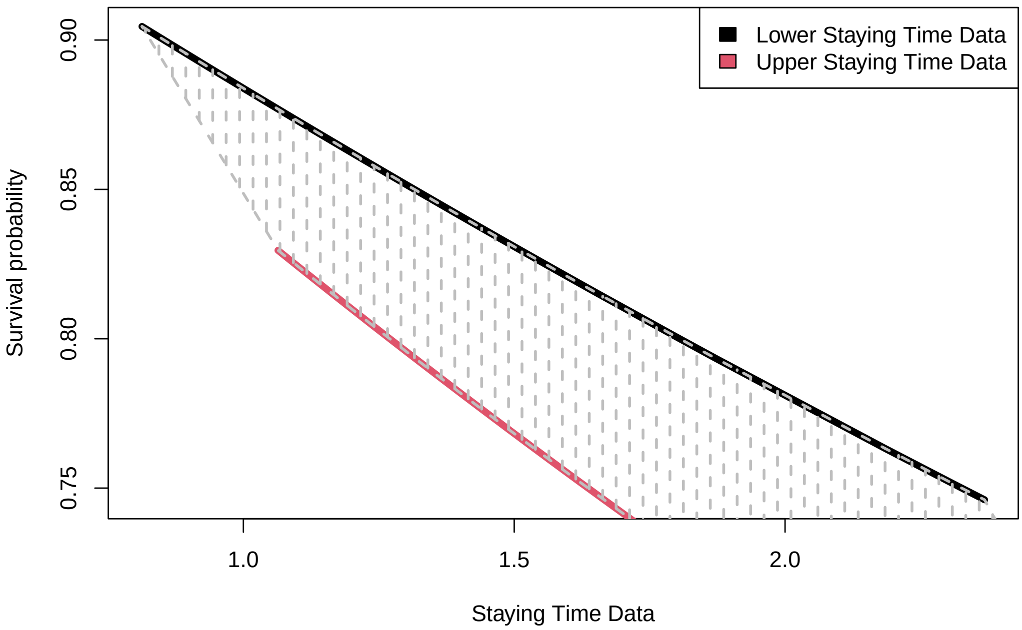

Figure 6: The survival curve plot for the NEPL distribution under MLE method

datasets and real-world data that contains uncertainties, ambiguity, or imprecision.

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