Electronic Journal of Applied Statistical Analysis
EJASA, Electron. J. App. Stat. Anal.
http://siba-ese.unisalento.it/index.php/ejasa/index
e-ISSN: 2070-5948
DOI: 10.1285/i20705948v17n3p703

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15 December 2024

Electronic Journal of Applied Statistical Analysis
Vol. 17, Issue 03, December 2024, 703-734
DOI: 10.1285/i20705948v17n3p703

A premium structure for a pandemic insurance policy

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This paper aims to establish an actuarial model tailored for insurance companies offering contracts covering pandemic events. The corresponding premium is determined based on both the probability of contagion and subsequent outcomes for insured individuals. We assume a one-year insurance coverage period, during which policyholders are exposed to the risk of contracting the virus. A random variable for contagion is defined, following a theoretical geometric distribution. Variable benefits are provided depending on the insured individual's "status" resulting from infection outcomes, with a lump sum payment offered in case of death. The duration of each "status" is modeled by a Gamma distribution. Utilizing these assumptions, a fair premium for the policy is estimated.

The risk analysis of the proposed policy involves quantifying benefit volatility, establishing a specified confidence level, and conducting threshold analysis using Markov's Inequality to determine the probability that benefits exceed a certain value. Additionally, we discuss the insurer's decision to customize the policy "prior" by applying a uniform premium to the entire population before applying a safety loading. Numerical applications are explored using weekly reports on the COVID-19 epidemic from the Istituto Superiore della Sanità (ISS).

keywords: COVID19, Insurance, Fair premium, Vaccinated and unvaccinated risk profiles.

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ISSN: 2070-5948
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1 Introduction

Uncertainties and risks are inherent in our world, impacting individuals, families, businesses, and countries. While it may not always be possible to prevent unfortunate events, the financial sector has developed products to mitigate losses by providing financial resources. Insurance, as a financial product, helps reduce or eliminate the costs and consequences of various risks. Insurers play a crucial role in enabling individuals and firms to share the risks they face Wardman (2020). Currently, factors such as increasing average life expectancy, technological advancements, and changes in disease patterns have led to a rise in private health insurance. In recent times, we have witnessed the emergence of several epidemics, including Middle East respiratory syndrome coronavirus (MERS), severe acute respiratory syndrome (SARS), bird flu, swine flu, and so on.

Since late 2019, the world has been grappling with the COVID-19 outbreak caused by the virus named COVID-19 (Al-Omari, 2024; Franchetti et al., 2023; Alradaideh et al., 2022). The dynamics of this pandemic have indeed been notable for the rapid spread of the disease. Additionally, there exists a vast body of literature demonstrating the significant impact of pandemics across various domains such as health, sociology, politics, and economics. It's undoubtedly a significant challenge for humanity. The resulting restrictions and measures taken to combat the crisis have had a significant impact on travel, business operations, and supply chains. Individuals and businesses are incurring substantial costs and losses, raising questions about available insurance coverage and the potential need for policy interventions in the future. Insurance serves as a critical financial tool during extreme events like disasters, providing economic protection to exposed populations Wister and Speechley (2020). Consequently, the COVID-19 epidemic, which has affected billions of lives, has started to reshape not only the insurance industry but also numerous other industries worldwide. During this challenging period, many countries have covered the full treatment costs for COVID-19 illness under private health insurance policies.

In our paper, we focus on analyzing the performance of an insurance policy designed to cover pandemic risk, which was developed and introduced to the market during the COVID-19 pandemic. We specifically address this because prior to the pandemic, we believe that this type of risk did not receive adequate attention, but it suddenly became significant following the outbreak of COVID-19. Typically, policies of this nature are engineered and sold outside of pandemic scenarios. The uniqueness of our paper lies in the Italian experience, where insurance companies swiftly created and offered policies to cover COVID-19 after the outbreak. Our paper draws inspiration from this context, inviting the reader to immerse themselves in it to understand a path similar to that followed by Italian insurance companies.

The objective of this paper is to propose an actuarial model for an insurance policy against the risk of a pandemic outbreak, specifically the risk of getting infected by the disease. It may initially appear illogical to offer and underwrite such a policy, as pandemics are widely recognized as distinct from regular diseases typically covered by insurers. The nature of a pandemic, including the frequency of infection and the severity of the disease, leads to different financial consequences compared to regular diseases

International Actuarial Association (1998).

Wrapping up the entire description of our targets, the goal is to define an insurance contract that has the scope to act like a health insurance covering sickness days and death events. This policy is useful because it provides the possibility to recover from the financial losses due to the sick status. In a more precise manner, we are considering losses as all the costs incurred together. In case of death, the contract aims to partially compensate for the income losses resulting from the insured agent's death.

However, this framework is based on the understanding that pandemic outbreaks are not uncommon events, and it is prudent to consider financial solutions to mitigate the financial losses resulting from infection. Pandemics, like the COVID-19 outbreak, exhibit a wave-like pattern of infection due to the nature of the disease and the rapid response of government institutions in containing it. An example of this response is the development and widespread distribution of vaccines to the population, which can alter the pattern of infection waves and subsequently impact the financial consequences for individuals.

In this work, the COVID-19 outbreak is used as an example of a pandemic outbreak, utilizing relevant data and the wave-like nature of the disease. However, it is important to note that this model can be adapted to any type of pandemic disease, considering its unique characteristics and the transmission dynamics from non-infected to infected individuals. The disease outcomes can vary, ranging from a single outcome (such as hospitalization and recovery over a specific duration) to multiple outcomes, as demonstrated later in the framework.

It is necessary to clarify the definition of infection outcomes in this work. Once a person is infected, the symptoms may vary, with some individuals being asymptomatic while others experience varying degrees of disease severity. The severity of the disease determines the appropriate care path for the infected individual.

In this paper, infection outcomes are referred to either as states or events. This is correct because once the infection event occurs, the infected person enters the infected state. Subsequently, there will be the infection outcome, which is an event where the infected person incurs in a given state. The reader is invited to follow this logic in order to understand later discussions where the infection outcomes are considered both events and states.

In the COVID-19 example, there are four levels of severity, and thus care patterns:

- Home quarantine: No symptoms (but still with active and contagious infection). The ones considered are the declared ones such that are present in the administrative data, so asymptomatic people are not present in the data and then, in the model.

- Hospitalization: Symptoms' severity now force the infected individual to take care by physicians in hospital

- Intensive Care: More severity of symptoms forces physicians to assign the infected individual in to a special care department to take care of it

- Death: Symptoms are so severe to cause the death of the infected person

The OECD OECD (2020) has released a very interesting initial assessment of coverage and gaps concerning the costs and losses related to the pandemic. Anyway, generally speaking, families in Italy are well covered for losses relating to death, health costs, and medical bills in illness cases, but they may be very interested in insurance-type coverage related to their health due to the caring costs. In this context, this paper proposes an actuarial model dealing with the issue of insuring against the risk of contracting the COVID-19 virus. It is reasonable to think that a COVID-19 insurance policy could be interesting because in Italy employees undergo an economic penalty on their wages in illness cases that force them not to work. For autonomous workers, however, there is no economic coverage of salary in illness cases. For this reason, we believe that a COVID-19 insurance policy, which more generally falls under health insurance policies, could be interesting for the Italian market. In fact, this argument is very heartfelt and discussed in the business environment as well as in the whole society. It is possible to consult the relevant literature, and the reader is advised to consult the contributions of Levantesi and Piscopo in 2020 and 2021 Levantesi and Piscopo (2020), Levantesi and Piscopo (2021). Focusing on the financial business, it is possible to see that this pandemic has brought about a lot of debates about the challenges that insurance companies have to face in order to cover the epidemiological risk and, more specifically, provide workers with protection in case of job interruption.

The corresponding premium is quantified based both on the probability of contagion and the subsequent outcome for the insured. Referring to the designed coverage, a risk analysis is offered by investigating the benefit volatility, constructing a confidence level, and making a threshold analysis through the application of the Markov Inequality. It is a common practice in actuarial analysis to compute the probability that the benefit is above a specific threshold. Here we follow this approach and compute the probability that the insurer will pay more than a given amount α, as can be seen later. Moreover, the choice of the insurer to make a "prior" customization, either by applying the same premium to the entire population or a safety loading, is discussed.

The data available to us consist simply of frequentist observations regarding an individual's status in relation to a potential infection: either infected or susceptible, i.e., not infected. Concerning insurance coverage and the occurrence of contagion within it, it is important to specify that the probability of being infected on a specific day is independent of the event of being infected on another day of coverage, as it is assumed that the event could happen on any day within the coverage period. However, it is evident that for each day of coverage, the occurrence of infection is dependent on the absence of infection events in all preceding days. In other words, a geometric distribution is employed, where the infection event on day t represents the probability of that event happening after $t-1$ days without occurrence. Therefore, each day carries its own probability of infection, independent of other days, but each probability is contingent upon the condition that the infection event did not occur in the preceding days.

The paper is organized as follows. Section 2 describes the model. In section 3 the dataset is presented. Section 4 performs the analysis, while section 5 discusses the risk evaluation. Section 6 concludes. In order to make all the results reproducible, all the R scripts can be downloaded at `https://osf.io/vw6nu/`.

2 The model

The proposed model is composed of two different sections that are strictly linked to each other:

- the statistical procedure suitable for describing the dynamics of the COVID19 epidemic;
- an actuarial model for quantifying the fair premium of a COVID19 insurance policy.

Regarding the actuarial model, it is based on the structure for quantifying health insurance policy premiums Pitacco (2000). On the other hand, the description of the epidemic dynamics is based on the traditional epidemic SIR model presented in Kermack and McKendrick (1927) Kermack and McKendrick (1927) for analyzing the dynamics of the epidemic and its subsequent releases proposed in He et al. (2020) He et al. (2020) and Zhou et al. (2004) Zhou et al. (2004). A very useful model to understand the relationship between actuarial premium and epidemic dynamics is the one of Lefevre et al. Lefèvre et al. (2017), where the authors used the removal rate, the dimension of epidemics, and infection status time to describe the dynamics of epidemics reliably. In our framework, we use a different approach that uses a cost/frequency model to describe, for each observation time, the current status of the entire population.

2.1 The stochastic approach

The assessment of the epidemic dynamics takes into account the data presented by the Istituto Superiore della Sanità (ISS) in its weekly report, which provides day-by-day information on the COVID-19 epidemic. For detailed data, refer to the dataset named PaperRDataset.csv in the OSF data storage (link in the Abstract). The observation period for the analysis is from 14 July 2021 to 09 January 2022. The model used is a stochastic compartmental model without demography, as described in Greenwood and Gordillo (2009) Greenwood and Gordillo (2009), consisting of six "boxes" representing different nature states. Four of these states are combined into a single "box" representing the overall nature state. The probability of infection is calculated using the classical approach based on Markov's probability property Markov (1954), considering the entire reference population. The same approach is applied to compute the probability of a specific infection outcome, considering the infected individuals as the reference population.

The probability p^I represents the probability of being infected conditioned on the fact that the person is susceptible. So, by the laws of probability, we have $p^I = \frac{Pr(I|S)}{Pr(S)} = \frac{\frac{I}{S}}{\frac{S}{N}} = \frac{I}{N}$. This demonstrates that the probability of being susceptible is clearly $p^S = \frac{S}{N}$ and the probability of being infected is $p^I = \frac{I}{N}$.

We can assign a specific set of individuals to each state considered in the model. The model graph is depicted in Figure 1. In Table 1, N represents the number of individuals belonging to the reference population, S stands for the number of individuals who are

Table 1: Infection and related outcomes probabilities

Non infection S	Infection I	Home quarantine HI	Hospitalization H	Intensive care IC	Death D
$p^S = \frac{S}{N}$	$p^I = \frac{I}{N}$	$p^{HI} = \frac{HI}{I}$	$p^H = \frac{H}{I}$	$p^{IC} = \frac{IC}{I}$	$p^D = \frac{D}{I}$

susceptible to the infection but not yet infected, I represents the number of infected individuals, of which H refers to those who are hospitalized, HI to those in home quarantine, IC in intensive care, and D represents the number of deaths.

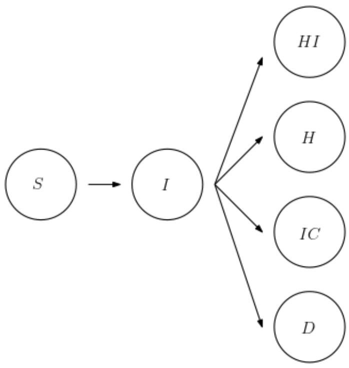

Figure 1: Multi-state model of infection dynamics

The following relations hold:

$$p^I = \frac{Pr\{I\}}{Pr\{N\}} \tag{1}$$

$$p^Z = \frac{Pr\{Z\}}{Pr\{I\}} \Rightarrow \sum_Z p^Z = 1 \tag{2}$$

The state S represents individuals who are susceptible to getting infected by COVID-19, while I represents those who are infected by COVID-19. The set I can be further divided into four subsets: $I := \{HI, H, IC, D\}$, where HI represents individuals who are in home quarantine, H represents hospitalized individuals, IC represents individuals in intensive care, and D represents deaths. These, as presented before, are states, and it is desired to consider each state as a set of all individuals that incur in that state. This concept is illustrated in figure 2.

The occurrence of an infection leads to one of the four possible outcomes. Therefore, the sum of the probabilities of these outcomes is equal to 1, indicating that once an individual is infected, they will fall into one of the four subsets of the infected individuals

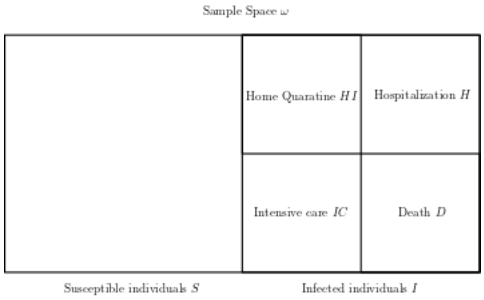

Figure 2: Sample space representation

set. It is important to note that all the possible outcomes are considered to be affected by COVID-19, so this equation holds only when a person gets infected.

About asymptomatic cases: The data only shows the number of people who are infected and subsequently hospitalized, in intensive care, or deceased. The only thing we can do is estimate the number of infected people who do not fall into these categories. However, further distinctions are not possible because administrative data does not account for these types of infected individuals.

This model considers time as the fundamental parameter, as in each observation time, there is the calculation of the probability that a person in the actual population is susceptible and then the probability, still in the actual population, to get infected. Thus, we can obtain a time series of the variables and, consequently, of the premium, which changes based on the current pandemic conditions in each observation time. In each data observation, there is a specific probability to get infected, and then there is a premium quantification that will be updated in the next observation time based on a change in the pandemic condition.

Next, we consider the probabilities of each infection outcome, except death, by referring to Equation 2. By calculating these probabilities for each observed data, corresponding time series can be obtained. For every infection outcome, except death, a periodic payment is defined until the end of the recovery. Let r_Z represent the single payment, assumed to be constant across the recovery days in a specific Z state, and $E[r_Z]$ represent the total expected payment throughout the individual's recovery period.

To calculate the expected insurance benefit, the expected value of the recovery days is multiplied by the benefit r_Z for each recovery day. In other words, r_Z stands for the periodic benefit paid by the insurer in case the insured person incurs in a state Z. Additionally, a lump sum payment is assumed in the case of death outcome, resulting in $E[P] = E[r_Z]+K \cdot p^D$. Here, $E[P]$ represents the expected total benefit, $E[r_Z]$ represents the expected payment for non-death outcomes, and K represents the lump sum benefit for death cases.

2.2 The actuarial model

The computation of probabilities in Table 1 allows us to quantify the fair premium for the proposed COVID-19 insurance policy. This is done by considering both the probability of getting infected and the probability of experiencing one of the four possible infection outcomes: hospitalization, intensive care, and death.

The insurance coverage is assumed to last for one year, during which the policyholder is exposed to the risk of contracting the virus. The occurrence of an infection event (I) is assumed to follow a daily Bernoulli distribution with a parameter p^I, which represents the probability for a susceptible person to get infected. The theoretical distribution of the contagion probability follows a geometric pattern for each day of coverage. Therefore, the probability dynamics $p^I(t)$, where $t = 1, 2, 3, ..., n$ and n is the coverage period (365 days), are estimated. Of course, once the infection event happens, it will lead to one of the outcomes explained before. The event "infection in day t" with $t = [1, 2, ...]$, are independent and have the same probability p^I, and the stochastic day of first infection follows a geometric distribution with parameter p^I, i.e., the probability to be infected for the first time in day t, denoted by $p^I(t)$, is defined in Equation 3.

$$p^I(t) = (1 - p^I)^{t-1} \cdot p^I \qquad t = 1, 2, 3, ..., n \tag{3}$$

The probability of getting infected at time t is an absolute probability that a susceptible person transitions into the infected state, as in the model presented in Section 2.1, starting from the susceptible state. Everything starts with the probability to be in the state I or in S. Then, both are absolute probabilities that a person incurs in the relative state. The infection probability follows the nature of an absolute one. For this reason, in each observation time, we have a share of the population that is infected and the residual one susceptible.

The policy's benefit is expressed in monetary units, without specifying a particular currency. A variable benefit is offered, depending on the "status" of the insured due to the outcome of the infection, and a lump sum payment is provided in the event of death. The number of days the insured stays in a single "status" is assumed to follow a Gamma distribution. For every infection outcome, except death, the expected benefit $E[r_Z]$, where $Z \in \{HI, H, IC\}$, is calculated. Symbolic values for the benefits related to each infection outcome are defined and listed in Table 2. For the non-deadly outcomes, the benefits are intended as periodic, while for the death outcome, it is a lump sum one. Here, Z stands for one of the possible infection outcomes realized once a person gets infected, excluding the one of death. In some sense, Z could be interpreted as a subset of I such that $I \supseteq Z$. According to Italian actuarial practice, we set $K = 3000$. The benefits related to each infection outcome are listed in Table 2. For the death case, a lump sum payment of D is made. If it is considered that $E[r_Z] = E[r_{HI}] * p^{HI} + E[r_H] * p^H + E[r_{IC}] * p^{IC}$, such that $E[r_Z]$ stands for the weighted sum of the different non-deadly infection outcomes, the expected payment $E[P]$ can be written as follows:

$$E[P] = E[r_C] * p^{HI} + E[r_O] * p^H + E[r_{IC}] * p^{IC} + K * p^D \tag{4}$$

Table 2: Benefits related to the infection outcomes. The first three are due for each recovery day.

Home Quarantine	Hospitalization	Intensive Care	Death
1	1.5	2	3000

The data source is ISS - Istituto Superiore di Sanità. This dataset expresses only the two states of infection (infected or susceptible) but does not express any information about the reinfection event. It is important to note that these data are used for the probability set observable on a weekly basis. However, for the purpose of constructing the insurance coverage period, the probabilities are considered on a daily basis, using the probability set computed for each observation week.

In Italian actuarial practice, death benefits are typically higher compared to the expected total amount of periodic benefits paid in case of non-deadly outcomes, due to the severity of the consequences associated with the death event. The other benefits are designed to reflect the increased seriousness of non-death outcomes, which involve several recovery days. The expected payment, denoted as $E[P]$, is assumed to be constant for each coverage day.

To assess the fair premium $V_0(t)$, $E[P]$ is multiplied by the corresponding infection probability $p^I(t)$ and discounted at a technical annual rate i.

$$V_0 = \sum_{t=1}^{365} E[P] \cdot (1+i)^{-\frac{t}{n}} \cdot p^I(t) \tag{5}$$

3 The dataset

The data available to us are obtained from the regional platform and communicated to the Italian Government Health Authority on a daily basis. These data include information on the number of infected individuals, hospitalized patients, and people in intensive care. The dataset takes into account the transitions between different states and the possibility of re-infections. For further details, please refer to the file named PaperRDataset.csv, which is available on the OSF storage platform at `osf.io/vw6nu/`.

The dataset used in this paper corresponds to the weekly reports on the COVID-19 epidemic published by the Istituto Superiore della Sanità (ISS) from July 14, 2021, to February 9, 2022. The dataset includes information on three vaccination statuses: unvaccinated, incompletely vaccinated, and fully vaccinated individuals. However, for the purpose of this paper, we focus only on the unvaccinated and fully vaccinated individuals who have completed their vaccination cycle. Therefore, the reference population N consists only of these two sub-populations: $N = Pop_{unv} + Pop_v$, where Pop_{unv} represents the size of the population share that has not received any vaccination, and Pop_v represents the population share that has completed a full vaccination cycle. The reason for excluding individuals with an incomplete vaccination cycle is that their number was relatively small compared to those who either completed the vaccination cycle or had no vaccination at all. This is because the incomplete vaccination status typically lasts for a period of fourteen days, and for the purposes of this analysis, we have focused on individuals who have either completed their vaccination or remain unvaccinated. In Figure 3, it is possible to see the dynamic of the vaccination plan across time. It clearly exhibits the increasing of the total population's share for vaccinated people in opposition to the reduction of the one for non-vaccinated.

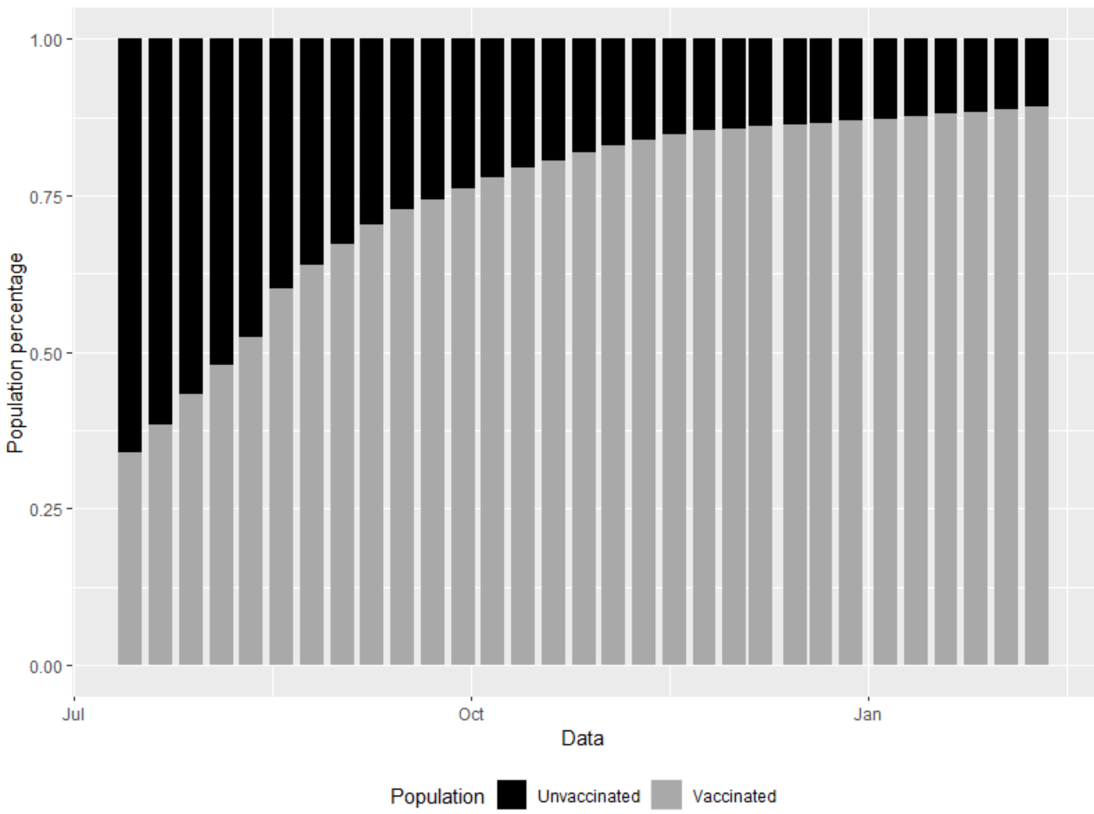

Figure 3: Population partition by vaccination status

Regarding the recovery days related to non-death outcomes, the mean of the hospitalization and intensive care days is obtained from Azienda Sanitaria Toscana Gemmi et al. (2021), and the data are available in the file PaperRDataset.csv on the OSF.io data storage (link in the abstract). The mean of the recovery days related to home quarantine is provided by the Health Ministry (2021). Azienda Sanitaria Toscana has collected data on the average time of stay in each state for non-death cases, while the Istituto Superiore Sanità (ISS) Istituto Superiore di Sanità (ISS) (2022) only provides the number of individuals belonging to each state.

The gamma distribution is assumed for the number of days in each recovery trajectory based on the outcome of the infection, with a shape parameter of 2. In Figure 4, you can see the shape of the gamma distribution, where the x-axis represents the number of days as the recovery period (and thus the number of daily benefits to be paid to the infected insured person). The y-axis represents the corresponding density, differentiated by different line types that specify the various outcomes, as shown in the legend.

This approach considers a probability distribution with a single tail, specifically the right tail, for a higher number of recovery days, with the probability mass concentrated on the modal value. In Table 3, it is possible to see the measures of central tendency and dispersion of the recovery days distribution for each infection outcome considered in the framework.

Table 3: Statistics of the recovery days

Statistics	Home quarantine	Hospitalization	Intensive Care
Mean	14	11	17
Mode	7	5.5	8.5
Std. Dev.	9.9	1.78	12.02

Our analysis is based on the dataset provided by ISS regarding infections, hospitalizations, and deaths, as determined by the health authority. It takes into account various circumstances, such as the fact that a hospitalized patient may have been infected and may have experienced multiple infections during the observation period (although less likely during the coverage period). The focus of our analysis is on absolute frequencies observed in the population for each state, which are crucial for the calculation of insurance payments.

The data from ISS and the Health Ministry represent the entire Italian population, while the data from ARS Toscana specifically pertain to the population of the Toscana Region. However, these regional data can be considered representative of the entire Italian population and are used as reference data to describe the progression of an individual's infection.

It's important to note that the ISS data is updated on a weekly basis. However, once the frequencies are obtained for the observation period, we assume a coverage period of 365 days from the underwriting day. During this coverage period, the probability of getting infected on any given day is conditioned on the absence of infection events in the previous days.

Unfortunately, we were unable to conduct analyses considering the age of the insured as a discriminating variable due to the lack of relevant data. It would undoubtedly be both pertinent and highly interesting to extend the analysis while also taking into account the age of the insured. However, the absence of data did not afford us the

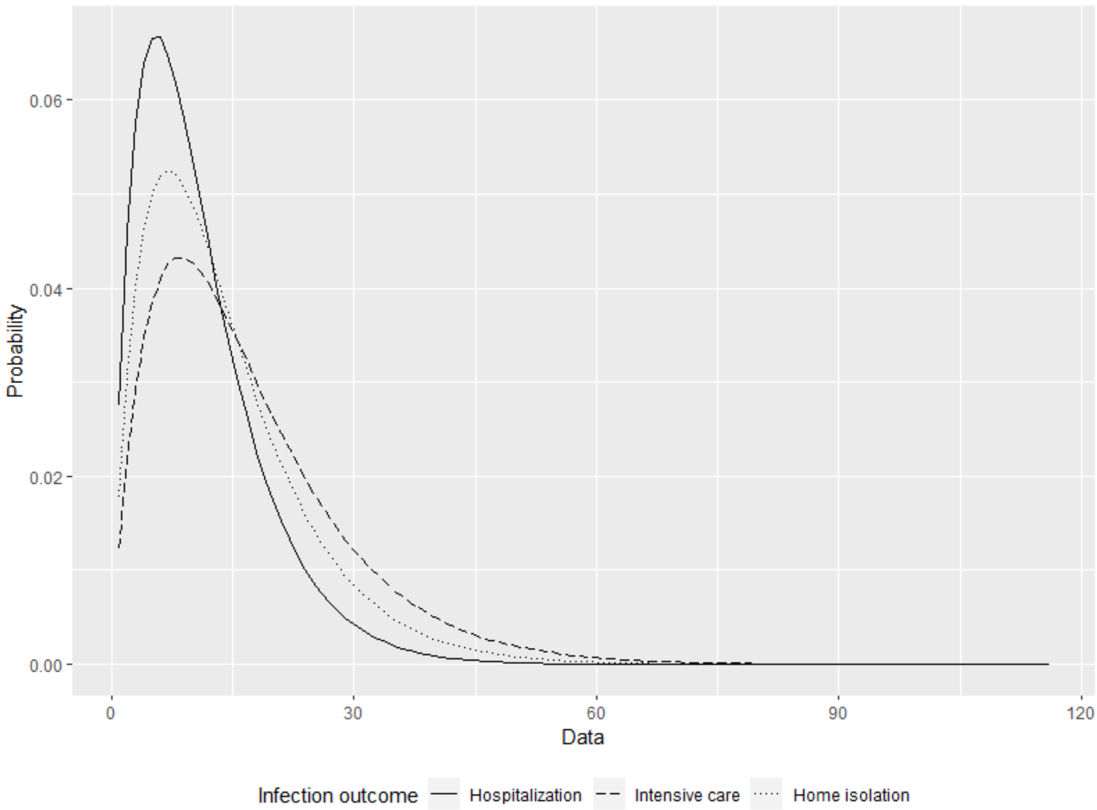

Figure 4: Theoretical gamma distribution of recovery days.

opportunity to do so.

4 Data analysis

Based on the provided dataset, we conducted an analysis to assess the evolution of infection probabilities, death probabilities, and expected benefits based on vaccination status. We calculated the fair premium for each class of insured.

We begin with the analysis of infection risk for each vaccination status, as shown in Figure 5. The results indicate that unvaccinated individuals have the highest probability of infection, making them the highest-risk group. Two infection waves were observed during the observation period: a moderate wave in August-September 2021 and a severe wave in January 2022. The impact of these waves was more significant for unvaccinated individuals, resulting in a higher increase in infection probability compared to vaccinated individuals.

Moving on to the outcome of infection, we focused on the worst-case scenario: death. The probability of death was found to be very high, with the highest probability observed among unvaccinated individuals, as depicted in Figure 6.

Please refer to the corresponding figures for a visual representation of the infection and death probabilities over time.

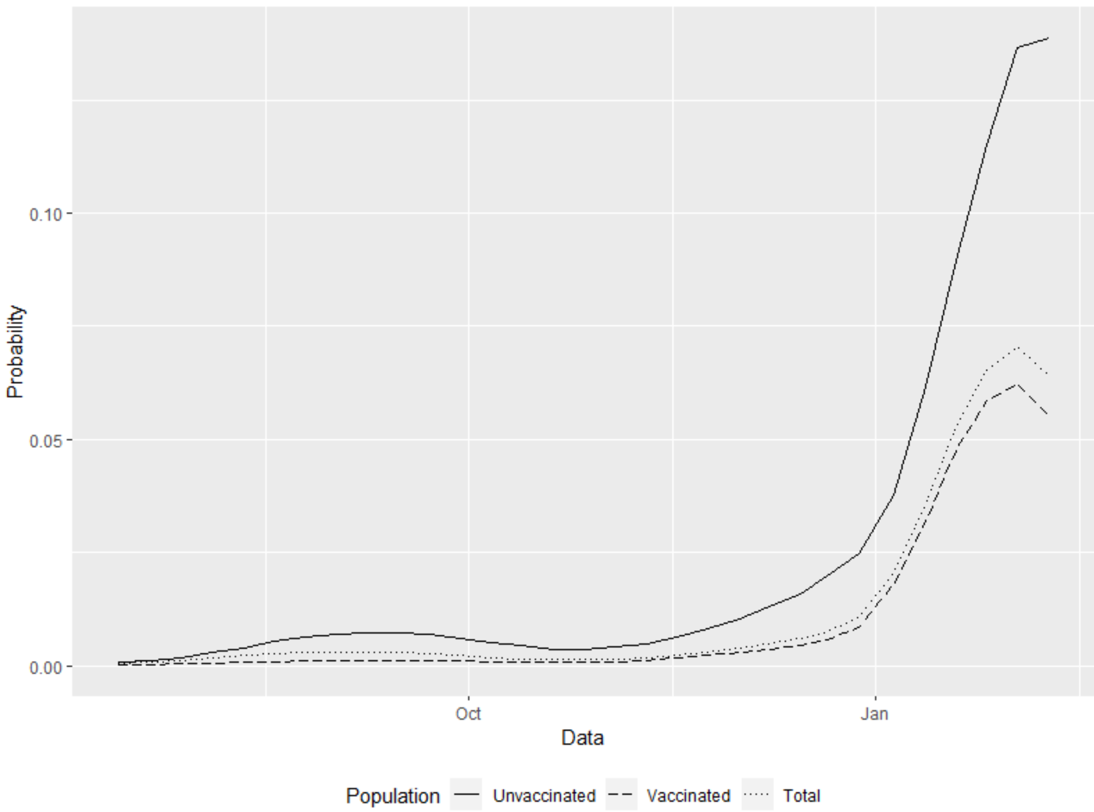

Figure 5: Time series of infection probability by vaccination status

This analysis demonstrates the positive impact of vaccination, resulting in a significant reduction in the risk of infection and related death outcomes. By considering both the observation data and the coverage days, we constructed an infection probability matrix. The surface plots in Figures 9 represent the distribution of probability mass between these two dimensions.

Table 4 presents the expected benefits associated with different infection outcomes, excluding the lump sum capital for death cases. The highest value is observed in the intensive care case, attributed to both the longer mean recovery days and the higher payment amount (as shown in Tables 2 and 4). Conversely, the lowest value is computed for the home quarantine case, reflecting the shorter expected recovery period.

Figure 10 illustrates that during the infection waves, a majority of individuals experienced home quarantine as a result of their infections. Consequently, a reduction in expected payments is observed during these waves, as the lowest benefit is expected to

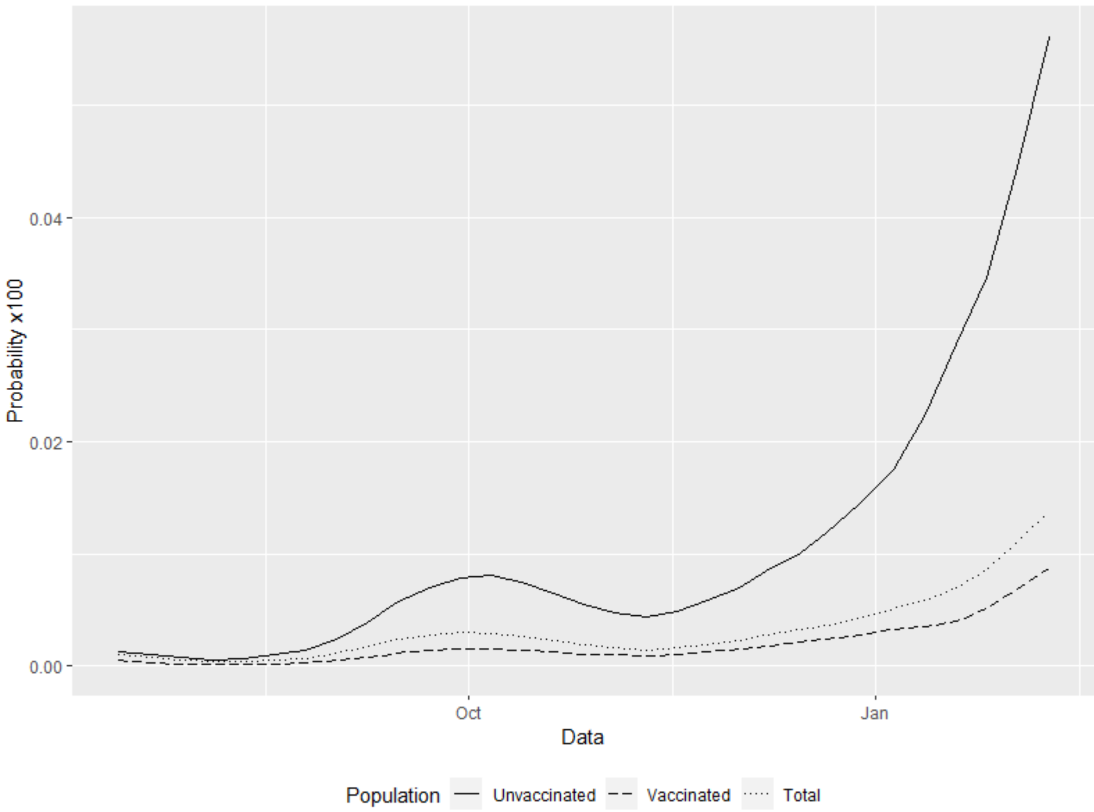

Figure 6: Time series of death probability by vaccination status

be paid.

Starting from October 20, 2021, the expected benefit for unvaccinated individuals surpasses that for the entire population. This is attributed to the implementation of the vaccination plan. Prior to this date, the probability to experience more severe infection outcomes, such as intensive care and death, was higher for vaccinated individuals compared to unvaccinated ones. This is due to the small share of vaccinated persons in the observed population prior to the aforementioned date. As the population's share of vaccinated persons increases over time, the results become more reliable, with unvaccinated persons showing a higher probability of experiencing severe infection outcomes.

Finally, based on the probability sets and the policy assumptions, we calculated the fair premium for vaccinated individuals, unvaccinated individuals, and the entire population. Figure 11 presents the dynamics of the premium, reflecting the risk profiles of both risk classes. The actuarial model confirms that the unvaccinated class has the highest fair premium compared to other vaccination statuses, highlighting their higher risk related to infection events and potential benefits.

It is interesting to note that pandemic evolution brings about weekly revisitations that

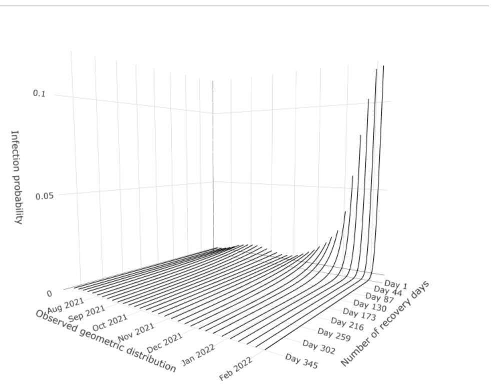

Figure 7: Vaccinated

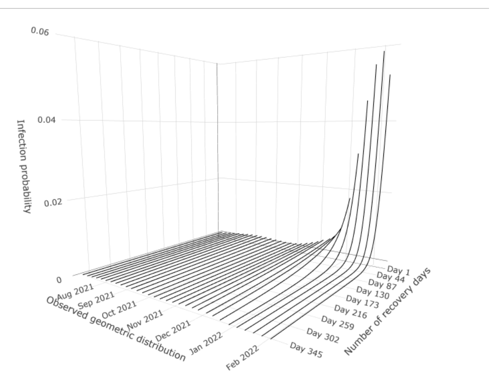

Figure 8: Unvaccinated

take into account the evolution of the considered variables. Clearly, it is necessary to underline that this kind of policy needs a continuous revisitation of premiums because they change in function of the different epidemiological phases and the several infection waves. It is natural to think about the high level of costs to face in order to follow this process, but in this way, the insurance company would preserve its solvency over time.

The insurance company can exploit the solidarity effect by practicing the same premium for the entire population regardless of the vaccination status of the individuals to whom the policy is offered.

Please note that these decisions should be made in consideration of various factors, including the regulatory framework, market conditions, and the specific context of the insurance company.

The premium increases during the first wave, as expected, and decreases outside of it, but during the second wave, the premiums are not getting higher as in the case of the first wave. The reason is that the infection probability mass distribution in the relative

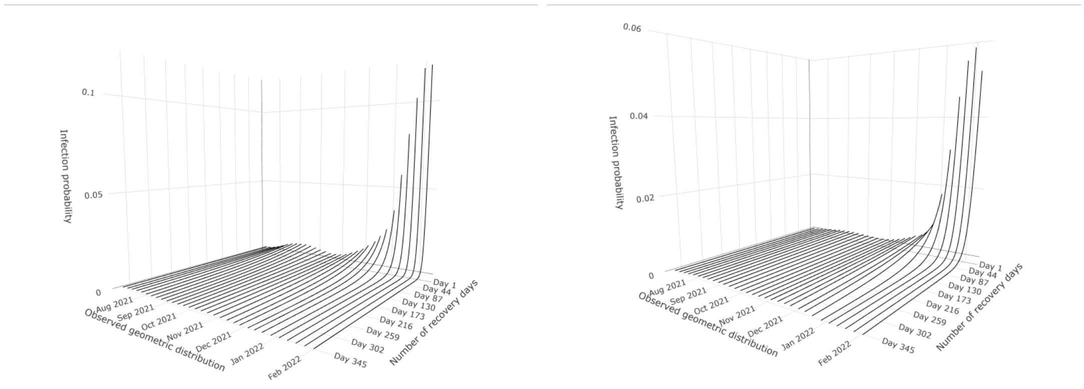

Figure 9: Surface plots of the infection probability matrix for unvaccinated and vaccinated

Table 4: Expected benefits for the infection outcome

Home Quarantine	Hospitalization	Intensive Care
14.02368661	16.54542718	32.03564112

matrix during the second wave is so strong that the mass is strongly concentrated in the very first days of the observation period. Then, very few discounts have a relevant weight, making, paradoxically, the premium to decrease. Observing these results, it is possible to state that the vaccination status has a relevant positive impact on the quantification of fair premium. In fact, people who are unvaccinated face higher risks both of getting infected and of experiencing a more serious infection outcome. The natural consequence is, obviously, that they should pay a higher premium in case of a prior premium customization.

Taking into account that the vaccination plan in Italy has been highly successful, the insurance company could think to practice a unique premium for the entire population of potential customers regardless of the vaccination status. To do this, it is necessary to apply a safety loading on the premium in order to cover the higher risk that comes from the share of the population that has not received any vaccination.

5 Risk analysis

The risk analysis of this policy includes several elements: benefit volatility, confidence intervals, and the assessment of benefits above a given threshold. Volatility is calculated by a weighted sum of the benefit variances conditioned by the infection outcome. In

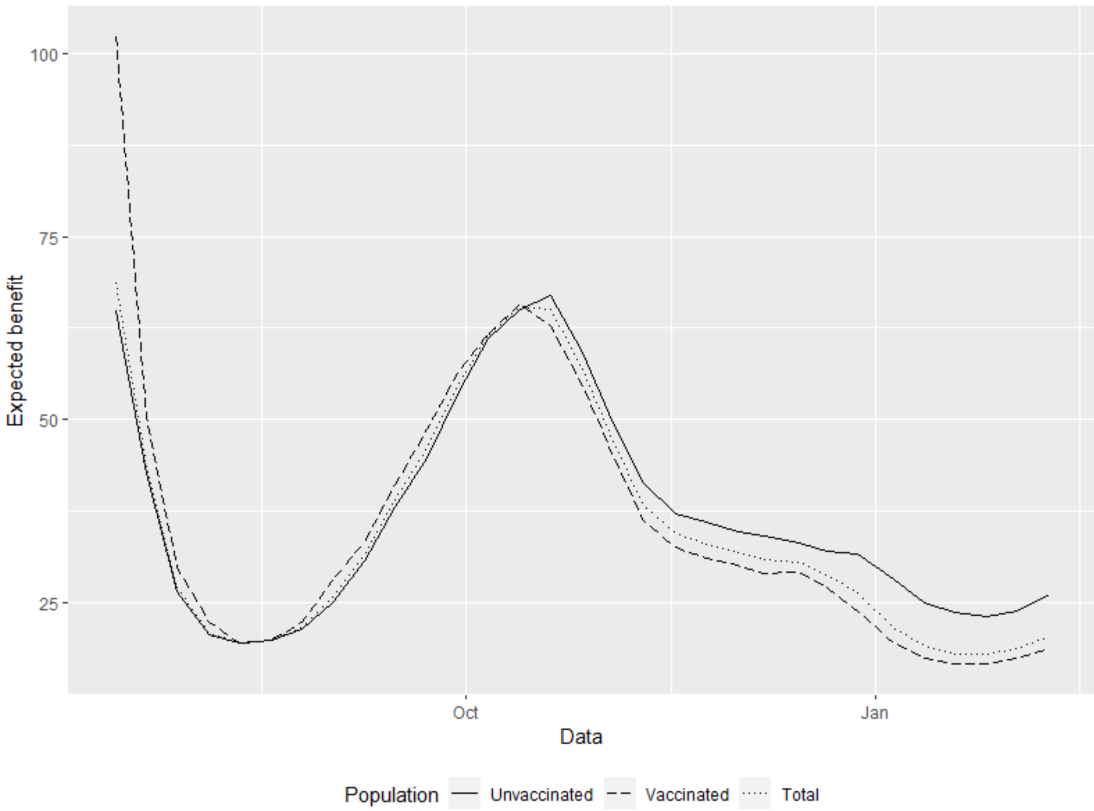

Figure 10: Time series of the expected benefits by vaccination status

Equation 5, the weighted sum of benefits variances, denoted by σ_Z with Z defined as before, represents the volatility of the total amount of periodic benefits paid by the insurer to the insured person. It is a function of the number of recovery days, each with its proper volatility. The total amount of benefits paid consists of the single benefit value multiplied by the number of days ($r_Z = b_Z \cdot n_Z$), where n_Z stands for the number of recovery days when the infection outcome Z is verified, and b_Z is the constant benefit paid each day of recovery when the outcome Z is verified.

The benefit costs follow the canonical identity present in health insurance policies, representing compensation for the cost of the disease rather than loss coverage in wages or incomes. It's reasonable to assume that the marginal cost of care is equal for all the population because, on average, the cost of medicines is standard across the country.

There are three variances considered, except for the death-case benefit, which has zero variance since it is a constant value. The dynamics of variances categorized by vaccination status can be observed, and it is evident that they decrease during the two infection waves. This reduction in volatility is due to the fact that during the waves, the probability mass of the benefit distribution is primarily concentrated on home isolation

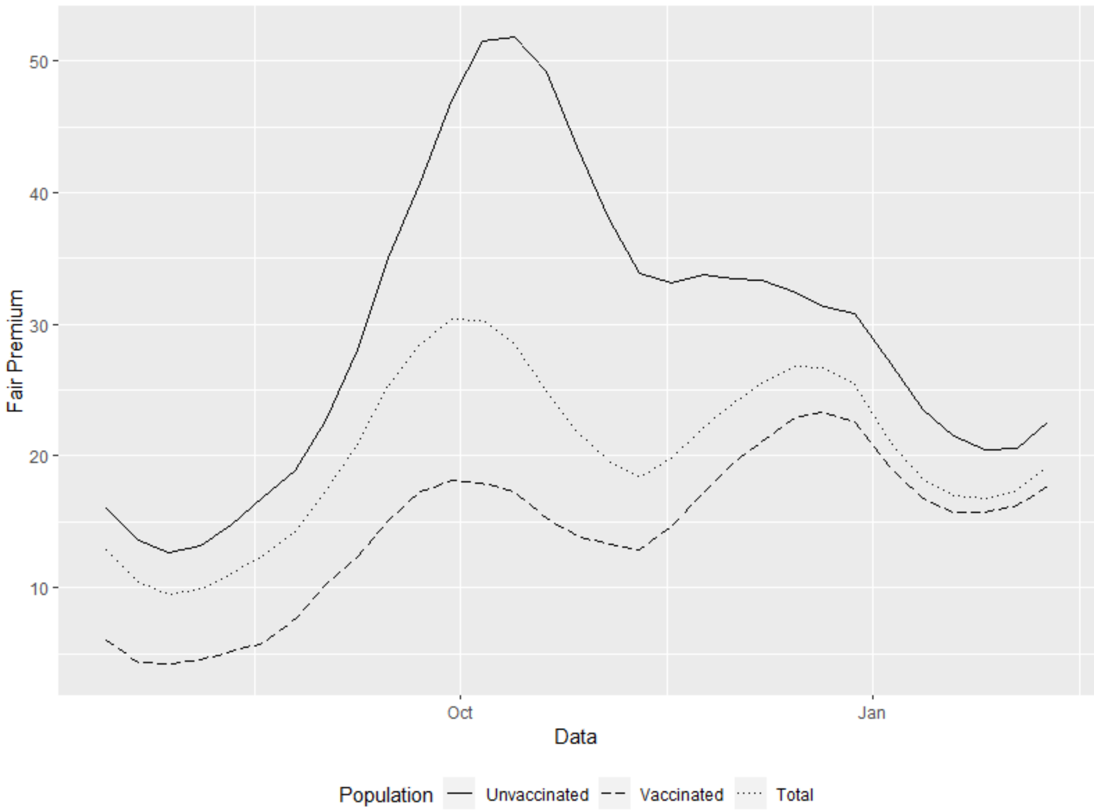

Figure 11: Time series (weekly update) of the fair premium by vaccination status.

cases. As a result, the company expects to pay a benefit that is not significantly different from the conditioned expected benefit of the home isolation outcome.

$$\sigma_P^2 = \sigma_{HI}^2 \cdot p^{HI} + \sigma_H^2 \cdot p^H + \sigma_{IC}^2 \cdot p^{IC} \qquad where\, \sigma_Z = b_Z * n_Z \tag{6}$$

Where b_Z, as said before, is a constant and n_Z is a variable of recovery days. The dynamics of volatility are presented in Figure 12.

Due to the irregularity of the benefit distribution, the assumption of a standard normal distribution is not used to calculate the confidence interval. Instead, Chebyshev's Theorem Pearson (1919) is employed to determine the threshold of the confidence interval. Specifically, the one-tail version of the theorem, known as Cantelli's Theorem Cantelli (1929), is utilized with the assumption of one tail in the right side of the probability distribution to establish a 95% confidence interval. The analysis focuses on the positive and negative deviations relative to 47.5% of each probability.

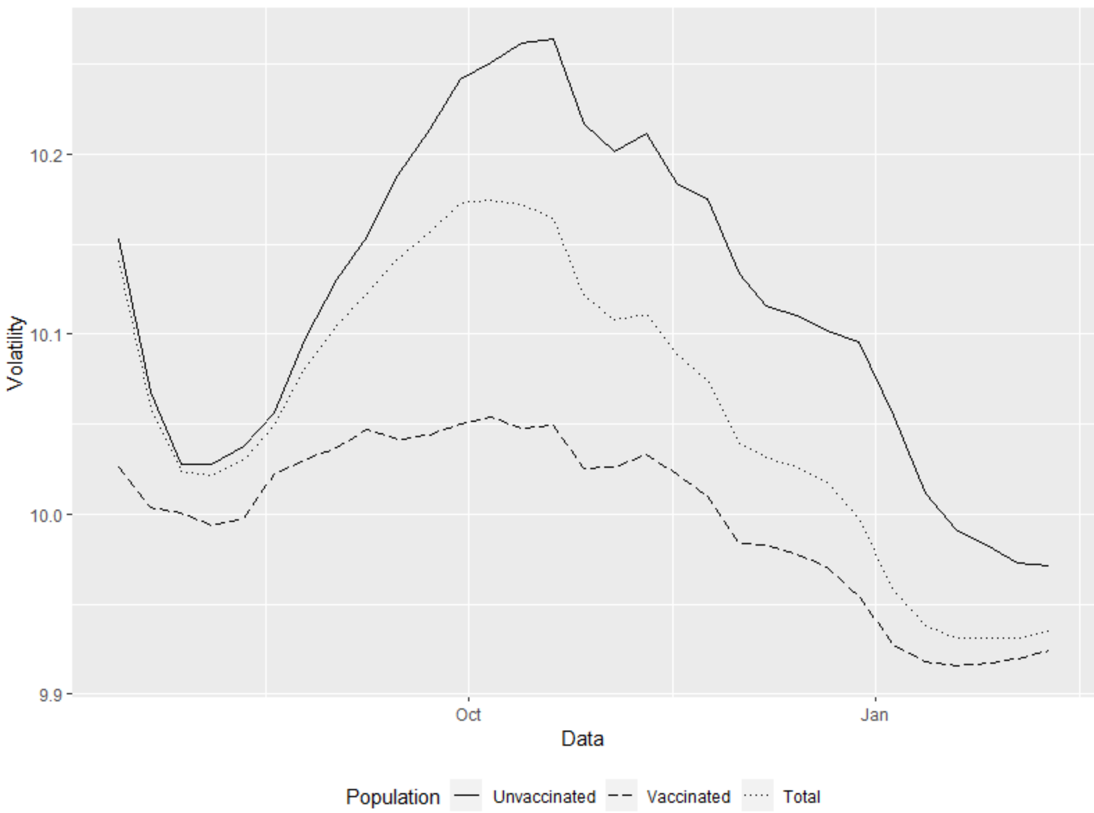

Figure 12: Time series of the benefit volatility

$$
\begin{aligned}
Pr\left\{P-E\left[P\right]\geq k\right\} &\geq 1-\frac{\sigma_P^2}{\sigma_P^2+k^2} \qquad \text{for } k>0\\
Pr\left\{P-E\left[P\right]\geq k\right\} &\leq \frac{\sigma_P^2}{\sigma_P^2+k^2} \qquad \text{for } k<0\\
\text{Upper End} \Rightarrow \frac{\sigma_P^2}{\sigma_P^2+k^2} &= 0.475 \rightarrow k^{max}=\sqrt{\frac{0.525}{0.475}}\cdot\sigma_P\\
\text{Lower End} \Rightarrow 1-\frac{\sigma_P^2}{\sigma_P^2+k^2} &= 0.475 \rightarrow k^{min}=\sqrt{\frac{0.475}{0.525}}\cdot\sigma_P
\end{aligned}
\tag{7}
$$

The dynamics shown in Figure 13 are very interesting. The dashed line represents the mean of the benefit, while the solid line represents the threshold of the 95% confidence interval. During the infection waves, the amplitude of the interval increases. This is because the wave brings a reduction in the expected benefit and volatility, as explained before, but the probability of death increases, as does the probability of home isolation. Meanwhile, the probabilities of hospitalization and intensive care decrease. This means

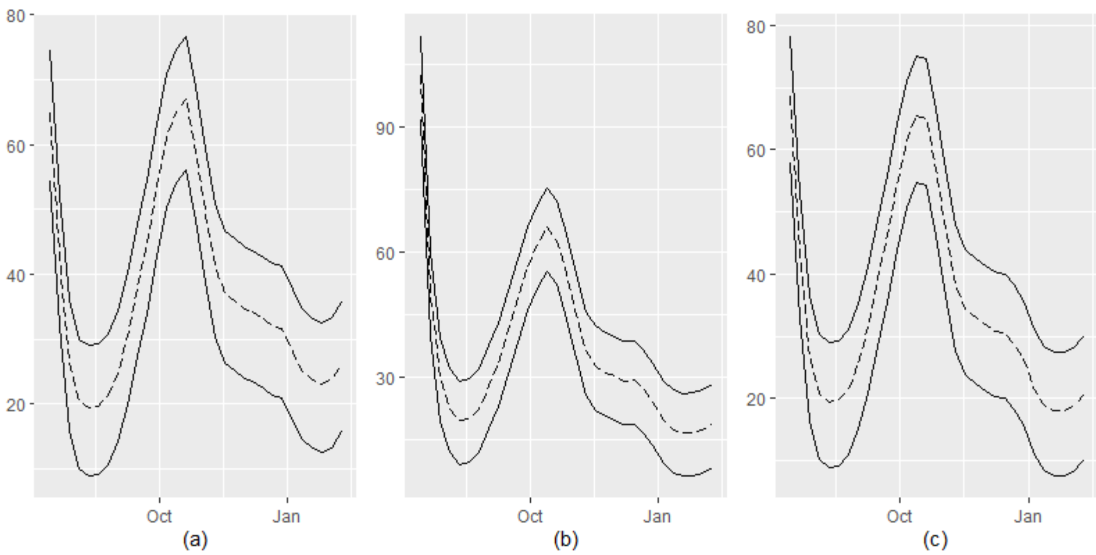

Figure 13: Time series of confidence intervals at 95% confidence level through Cantelli's theorem. Unvaccinated (a), Total (b), Vaccinated (c)

that the mean value decreases, but at the same time, there is an amplification due to the higher probability of death, resulting in a higher potential payment by the company.

Notably, for unvaccinated individuals, the amplitude is greater compared to vaccinated individuals, reflecting the higher probability of death for the former. Outside of the waves, this probability decreases, leading to a reduction in the amplitude during this period. However, there is an increase in the expected benefit due to the higher probabilities of hospitalization and intensive care, coupled with a decrease in the probability of home isolation.

In insurance companies, it is common practice to calculate the probability of paying a benefit above a certain threshold. Markov's theorem Ogasawara (2020) is used to calculate the probability of paying a benefit higher than a threshold value of $\alpha = 200$. As shown in Figure 14, during the waves, the probability in the right tail decreases due to the reasons explained earlier.

$$Pr\{P > \alpha\} = E[P] \cdot \alpha \tag{8}$$

5.1 Facing the impact of vaccination status.

The entire population is categorized based on their vaccination status, and as observed earlier, the corresponding premiums differ significantly, leading to important implications. When the insurance company decides to offer this insurance product, it must consider the estimation of benefits and the probability sets that vary over time and

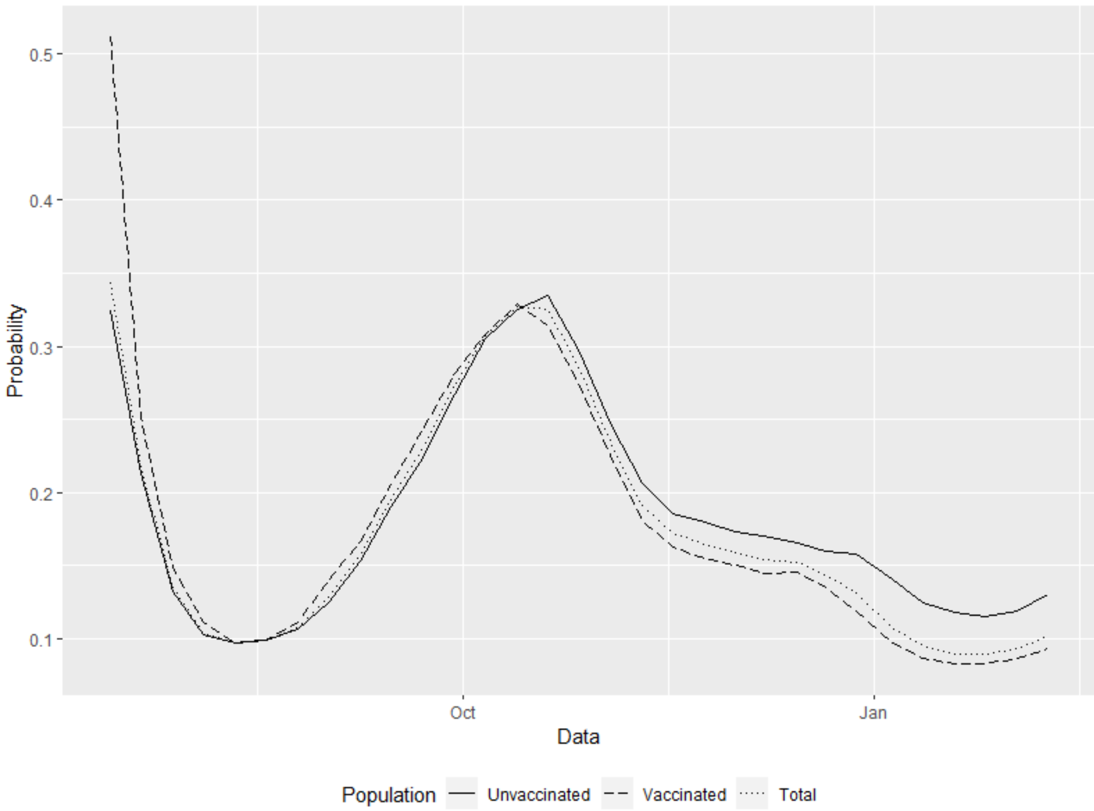

Figure 14: Time series of the probability to pay a benefit above the threshold of 200

vaccination status. Unvaccinated individuals have a higher probability of infection and are also more likely to experience severe outcomes, resulting in higher expected benefits. Consequently, the company may choose between two possible approaches: prior customization or applying the same premium to the entire population.

It is important to note that there are individuals with an incomplete vaccination status, who may be in a transitional phase from unvaccinated to fully vaccinated. These individuals may not need to be included in the model. Regarding the second solution of applying the same premium, it could be advantageous in a hypothetical case where the entire population consists only of unvaccinated individuals or only those with a complete vaccination status because there is total homogeneity in the population in terms of risk profile. It is efficient to apply the same premium to everyone without worrying about the heterogeneity in risk profiles that arises from the fact that in the population there are both vaccinated and unvaccinated people, hence both risk profiles.

As for the first solution, the company could exclude individuals in the unvaccinated category by including a clause in the contract stipulating that coverage starts only after the vaccination is completed, but only if the insurance company does not want to apply prior personalization of the insurance premium based on the individual's risk profile, including vaccination status. Table 5 presents the discounted actuarial values for the total population, considering solidarity between the two risk categories, and the premiums that individuals should pay based on their vaccination status under the prior customization solution. The table takes into account the benefits after 30, 180, and 360 days from the underwriting day. It is important to note that the benefit itself remains constant in each sum related to the total amount of benefits paid according to the total amount of realized recovery days, but it is multiplied by the probability of getting infected after the specified number of days. Therefore, the specified number of days represents the discount time horizon from the underwriting day. The table illustrates the presence of two sources of risk.

Table 5: Table of discounted actuarial values by vaccination status and coverage day for the infection event inside (26th January 2022) and outside (20th October 2021) the infection wave

	Status	Day 30	Day 180	Day 360
Outside the infection wave	Unvaccinated	0.21854367	0.12566557	0.06468893
	Vaccinated	0.04670816	0.0416107	0.03622261
	Total population	0.08245359	0.06749015	0.05307345
	Status	Day 30	Day 180	Day 360
Inside the infection wave	Unvaccinated	0.06720601	7.25E-10	2.00E-19
	Vaccinated	0.1581135	1.85E-05	3.56E-10
	Total population	0.15381837	6.27E-06	3.39E-11

The probability mass distribution throughout the coverage period is derived using the geometric distribution of the infection probability, taking into account the timing of the infection event. The consequences of the infection waves on the probability over the coverage period are evident. During the waves, only the initial discounts have significant relevance in terms of probability weights. For unvaccinated individuals, there is a higher concentration of probability mass in the early days of the coverage period compared to vaccinated individuals. This makes short-term discounts more relevant and necessitates a higher initial capital to mitigate the risk of infection in the short term. In other words, unvaccinated individuals present a higher risk of being infected, especially in the short term. Additionally, they have higher probabilities of experiencing more severe infection outcomes, leading to higher expected values and greater volatility of benefits.

For the entire population, the results lie between those of the vaccinated and unvaccinated individuals. It is interesting to note that during a severe infection rate, such as the one observed in January 2022, there is a significant increase in the mass of infection in the early days of the coverage period. This makes the discounts less relevant and, paradoxically, results in lower premiums. This phenomenon is observed across all three scenarios. It explains the dynamics of premiums and the risks associated with selling insurance policies. Unvaccinated individuals represent the class with the highest infection risk, and the insurance company must find a way to manage this risk.

Clearly, the impact of the solidarity solution needs to be carefully assessed by the insurance company. This work only demonstrates that there is a possibility of offering the same premium to individuals from both risk classes or applying specific premiums for each risk class. Therefore, the optimal solution is to implement prior personalization, where both vaccinated and/or unvaccinated individuals are covered with respective premiums.

5.2 Safety loading

One interesting approach to mitigate the loss risk associated with unvaccinated individuals is to calculate a premium for a policy sold to the entire population based on data from vaccinated individuals. This is feasible because, due to the vaccination plan, nearly the entire population is vaccinated. By doing so, an implicit safety loading can be applied with the objective of making the loaded premium equivalent to that of unvaccinated individuals when necessary. There are two potential ways to apply this implicit loading: by adjusting the infection probability and the benefit expectation.

At first glance, this solution may seem unusual, but it requires some clarification. Normally, safety loads are determined based on the risk analysis of the entire portfolio of insurance contracts offered and signed. However, in this context, the goal is to find an appropriate contract structure to mitigate the risk of excessive levels of risk in an environment where the insurer does not have precise information on the riskiness of each individual and is exposed to possible adverse selection phenomena Rothschild and Stiglitz (1976). This aligns with the argument presented in the previous subsection regarding the possibility of a single premium for the entire Italian market demand.

5.3 Infection probability matrix

In the first case, it is important to consider not only the occurrence of the infection event but also the timing of the event. During an infection wave, there is a higher likelihood that the infection event will occur in the early days of the coverage period. Conversely, outside of a wave, the probability distribution of the infection event is more evenly spread throughout the coverage period, increasing the probability that the event will occur in the later part of the period and decreasing the initial one. When the probability of infection is higher for the unvaccinated individuals, it may be appropriate to apply a loading on the probability for vaccinated individuals that accounts for the difference between the unvaccinated and vaccinated probabilities. Otherwise, if the gap between the probabilities is zero, it implies that there is no need for the company to apply a loading on it. The loaded probability to get infected, denoted as $(p_v^I)^*$, is the sum of p_v^I, the probability that a vaccinated person gets infected, and Δ_p^I, which represents the gap between the probability to get infected for a vaccinated person and the one related to an unvaccinated person. This sum, defined as $(p_v^I)^*$, is set to be equal to p_{unv}^I, thereby applying a safety loading on infection probability to capture the increase of risk due to non-vaccination.

$$(p_v^I)^* = p_v^I + \Delta p^I = p_{unv}^I \qquad \text{Where: } \Delta p^I = p_{unv}^I - p_v^I \text{ and } \Delta p^I \in [0; +\infty) \tag{9}$$

The company could consider using the probability matrix of the unvaccinated population instead of the one for the vaccinated population, while applying the gap on the probabilities for the vaccinated population. The surface graph showing the probability gap is exhibited in Figure 15.

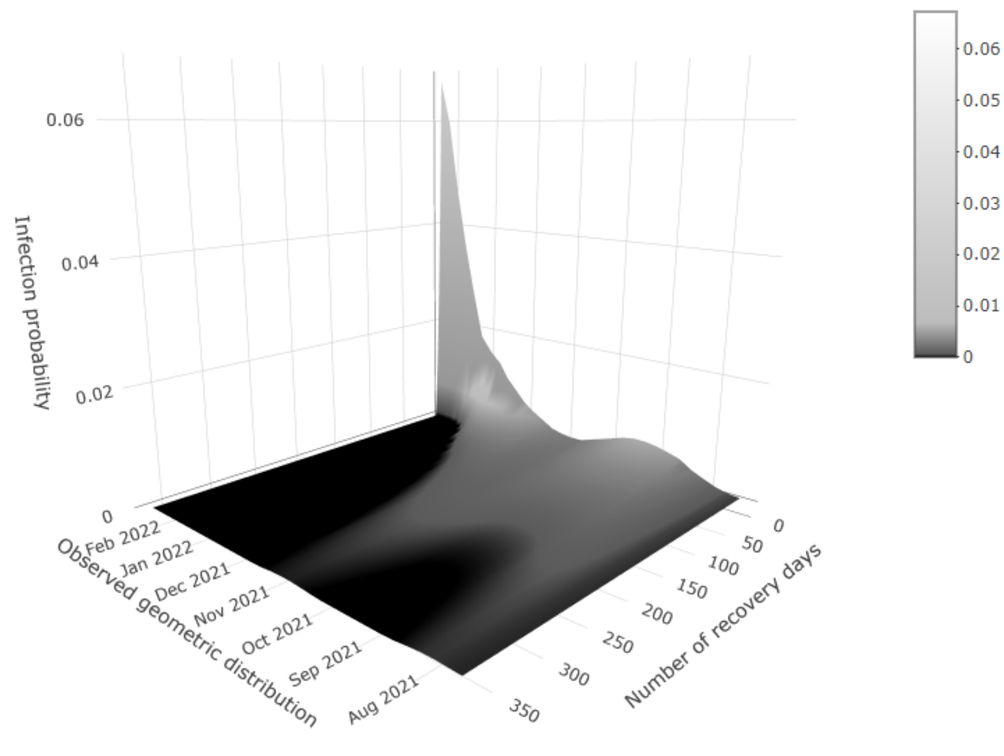

Figure 15: Surface plot of infection probability gap between unvaccinated and vaccinated persons

This can be done as long as the gap is positive. Essentially, the company would use the probability matrix of the higher risk unvaccinated population as a conservative estimate, incorporating it into the premium calculations for the vaccinated population. However, if the gap is non-positive, meaning that there is no significant difference between the probabilities of the two groups, there would be no need to apply a loading based on the gap.

5.4 Expected benefit

When applying a loading on the benefit expectation, the standard deviation principle Christofides (1998) can be used to determine how much to charge a vaccinated individual's expected payment so as to get the same expected payment as an unvaccinated individual. It is worth noting the dynamic behavior of the gamma parameter used in this calculation. The following equation shows the loaded expected benefit that takes into account the coefficient γ that measures the intensity of the loading according to the gap between the expected benefits related to unvaccinated and vaccinated persons, as defined in equation 11.

$$E\left[P\right]_v^* = E\left[P\right]_v + \gamma \cdot \sigma_v = E\left[P\right]_{unv} \tag{10}$$

So that:

$$\gamma = \frac{E\left[P\right]_{unv} - E\left[P\right]_v}{\sigma_v} \in [0, +\infty) \tag{11}$$

The gamma parameter serves as a measure of the intensity of the loading that the insurance company can apply. It ranges from zero to a positive value, where zero indicates that the expected benefit for the entire population is higher than that for the unvaccinated population, implying that no loading is applied. This is because we are in the context where the insurance company wants to capture, in the premium, the increase of risk related to unvaccinated customers. Therefore, when gamma is zero, it means that there is the same premium between vaccinated and unvaccinated customers due to null increase of risk. When gamma is positive, there is an increase of risk for the unvaccinated customer compared to the vaccinated one. Then, the insurance company wants to increase the premium according to this. It is excluded the situation when gamma is negative because it is related with the period when the vaccination plan was at the beginning. So very few people were vaccinated and the related infection probability is biased exhibiting itself greater than the unvaccinated one. Unvaccinated customer risk profile is necessarily different from the vaccinated one due to both vaccination nature and can not be otherwise.

Figure 16 illustrates this last concept. It is interesting to explore whether both the loading on the probability (for the infection event and its timing) and on the expected benefit (for the payment in case of an infection event) are applied. These loads are implemented on the fair premium for vaccinated individuals to derive the premium. The equation 12 presents the loaded premium as a function of the loaded infection probability, as defined in equation 9 and the loaded expected benefit, defined in equation 10.

$$\Pi = f\left((p_v^I)^*, E[P]_v^*\right); \qquad \Pi = f(p_v^I + \Delta p^I; E[P]_v + \gamma * \sigma_v) \tag{12}$$

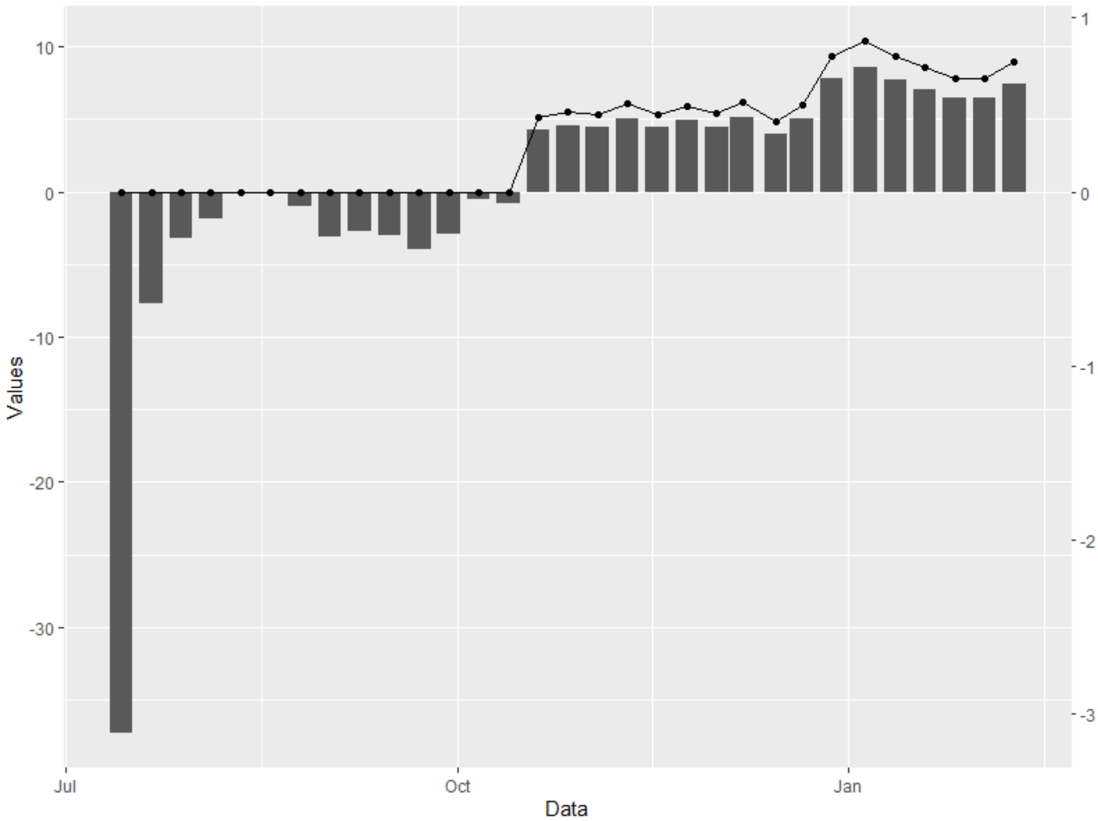

Figure 16: Gamma value time series

The result is the loaded premium for the policy offered to the entire population, based on data from vaccinated individuals, who represent the majority of the population, loaded in order to address the risk posed by unvaccinated individuals.

Figure 17 clearly demonstrates that vaccination leads to a lower premium compared to a loaded premium. The underlying dynamics are easy to comprehend.

As we progress through the observation dates, the probability gap diminishes, getting closer and closer to a null gap. This is consistently due to the exclusion of all non-reasonable scenarios where unvaccinated individuals have a lower infection probability compared to vaccinated ones.

As a consequence, only the initial discounts become relevant, leading to an increase in the loaded premium when compared to the fair premium for vaccinated individuals, starting in early November 2021. Prior to this, the burden caused by unvaccinated individuals is substantial throughout the coverage period, necessitating additional discounts to address their elevated infection probability.

The expected benefit remains unchanged until the second half of October 2021 because the expected benefit for unvaccinated individuals is lower, and the premium does

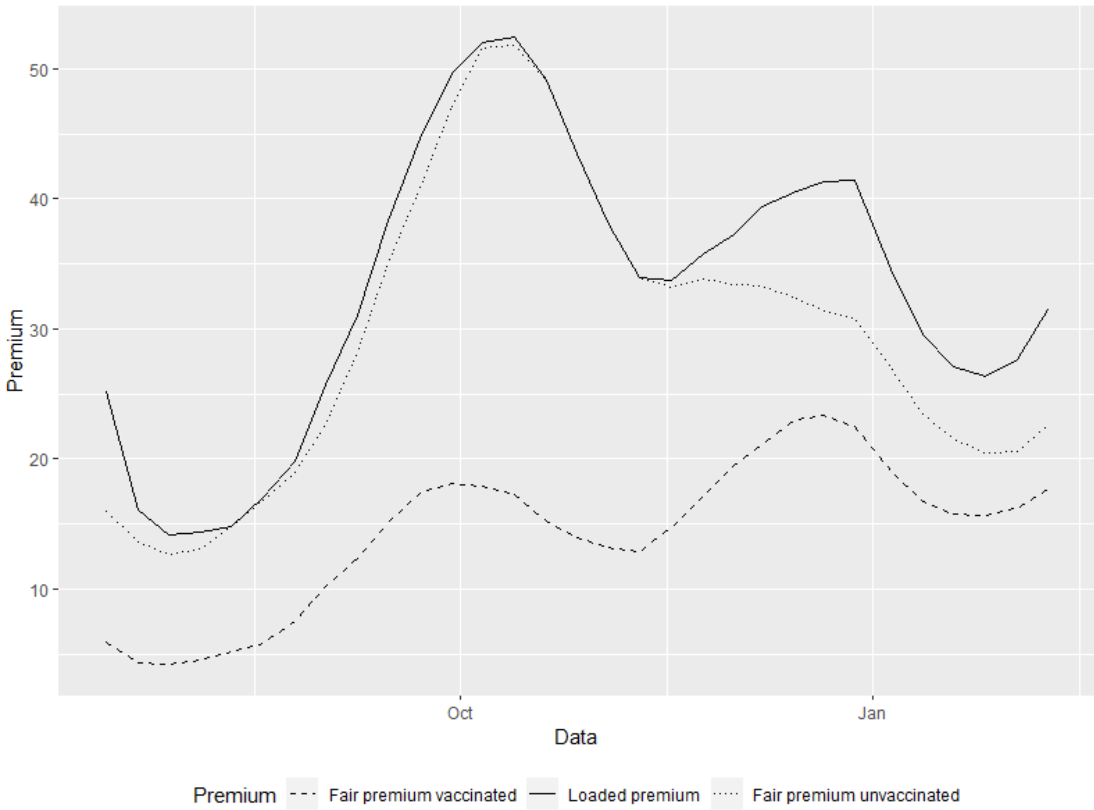

Figure 17: Surface plot of infection probability gap between unvaccinated and vaccinated persons

not require any loading. However, from that point onward, the gap becomes non-zero, requiring the application of a loading, determined by the gamma parameter, on the expected benefit of vaccinated individuals. This accounts for the increased risk of higher payments due to unvaccinated individuals. The loading on the expected benefit of vaccinated individuals is a function of gamma, which in turn is based on the gap between the expected benefits of both risk profiles.

As a result, the loaded premium is higher than both the fair premium for unvaccinated individuals and that for vaccinated ones. This is because we have excluded cases in which unvaccinated individuals have a lower risk of infection and consequently a lower expected benefit to be paid by the insurer. This is a direct result of the evolving success of the vaccination plan over time. In this method, the insurer is safeguarded against potential positive deviations from the expected benefits owed to unvaccinated underwriters. The key advantage of this approach lies in its ability to define a premium that comprehensively considers the entire spectrum of risk factors, ensuring the insurer's ongoing solvency.

The adverse selection theory outlined in Rothschild and Stiglitz (1976) suggests that the standard error principle method may dissuade vaccinated individuals from seeking insurance coverage, as they perceive the premiums to be unfairly high. However, this issue can also affect unvaccinated individuals, particularly at the beginning of the vaccination plan. Initially, the premium burden may fall on unvaccinated individuals, with a safety loading included to cover insurance expenses and potential deviations in benefit payments from expectations. Paradoxically, as the vaccination plan becomes more successful, the combined effect of these two loads causes the premium to significantly exceed the fair premium for unvaccinated individuals. Consequently, even unvaccinated individuals may find it disadvantageous to purchase the policy.

To address the impact of the vaccination plan effectively, the solidarity approach can be adopted, where the premium is the same for both risk classes. Within this premium calculation framework, the vaccination plan is considered at all stages. As a result, as the population transitions from unvaccinated to vaccinated status, the policy premium shifts from the unvaccinated fair premium to the vaccinated one, as illustrated in Figure 11.

After the occurrence of the second wave, we can observe that, thanks to the successful vaccination plan, the loaded premium widens the gap compared to the fair premium for unvaccinated individuals. This widening is primarily because the applied loading system does not consider all scenarios in which vaccinated individuals exhibit higher risk levels than unvaccinated ones.

Upon closer examination of the plot's line shapes, it becomes evident that the most significant impact of vaccination is on the probability of experiencing outcomes that are more severe than home isolation, in comparison to the simple infection outcome. As a result of this observation, the gamma parameter increases the loading, which is dependent on the volatility of benefits for vaccinated individuals. Furthermore, this benefit volatility decreases in the most recent observation data.

6 Conclusion and Future Research Line

We attempted to develop a premium structure that considers a disease with four possible outcomes, classified into two traditional models: death and survival with varying degrees of severity. However, for a more general model, insurers could easily adapt the framework by defining the potential outcomes of a particular disease. This allows insurers to establish a framework consistent with the concept of the disease, which can then be extended to integrate other outcomes related to survival. In a scenario where the risk of a pandemic (such as COVID-19) is perceived as remote and distant by the market, the demand for insurance policies covering such events is expected to be very low, as the perceived usefulness of policy subscription would be minimal. Indeed, in Italy, such policies were scarcely available before the pandemic; however, following the outbreak, there was a rapid and sudden surge in both demand and market supply for this type of policy. This could lead to a shift in pandemic risk perception and subsequently to a new market equilibrium, as well as an enhancement and strengthening of actuarial practices.

One advantage of this model is that it does not require the modeling of disease dynamics with a mathematical diffusion model but rather focuses on observing the number of people infected. The framework already considers this pattern in probability estimations. Thus, the only requirement is to gather data from previous pandemics and study them to determine the probabilities of death and survival outcomes. Another advantage is that insurers can start a new insurance policy based on a population partitioning strategy, as seen during the COVID-19 pandemic.

The first result derived from our model highlights the importance of considering a pandemic insurance policy. It is not reasonable to assume that coverage against pandemic-related illnesses is unnecessary. The dynamics of such diseases are significantly different from other illnesses, as mentioned in the abstract. A targeted approach is necessary to effectively address this type of pathology, providing valuable coverage to the insured while also ensuring solvency for the insurance company. It is crucial to recognize that including a pandemic-related illness in a general health insurance policy can result in much higher costs than initially estimated. By designing a specific insurance instrument for pandemics, a profitable tool for the company and an effective solution for the insured can be created.

An essential aspect of this framework is that the loading system is not an effective approach to address the risk gap between vaccinated and unvaccinated risk profiles. Instead, applying the solidarity principle is a better strategy. Under this approach, it is acknowledged that, due to adverse selection, vaccinated individuals may not be incentivized to purchase the insurance contract. However, it is also true that the premium reflects the diminishing population of unvaccinated individuals, resulting in a smaller aggregate expected benefit. Moreover, in the case an unvaccinated person underwrites the contract, the insurance company has a loading measure to get covered by eventual payment higher than expected.

In this context, vaccination is mandatory. Therefore, it is reasonable to establish a common premium. By offering reduced premiums to individuals who get vaccinated, there is a greater incentive to purchase the policy. The principle of solidarity then comes into play, with the benefits of unvaccinated individuals being financed by those who are vaccinated. The loading strategy presents a promising avenue for further research, contributing to the expansion of the existing literature, particularly in the field of pandemic risk insurance.

The strength of the implicit loading method lies in its ability to provide insurance coverage for the insurer against potential excesses in infection probability and expected benefits, which may arise from an unvaccinated risk profile of the underwriter. This is achieved through a loaded premium, which significantly increases compared to the premium for an unvaccinated individual. This increase is a result of the combined influence of these two risk factors.

Furthermore, it is important to note that the evaluations conducted focused on individual policyholders rather than a portfolio. The intention was to concentrate on the individual financial design of the policy, which can subsequently lead to portfolio-level assessments of insured individuals. The aim is to work on the relationship between the insurance company and the individual policyholder. Based on the individual's risk profile regarding the possibility of infection and the severity of potential outcomes, the policyholder is faced with the decision of whether to proceed with the contract or not.

This model has indeed laid the foundation for the design of a financial insurance policy capable of addressing the financial consequences of potential pandemic-related illnesses such as COVID-19. This model allows for the consideration of any type of pandemic-related illness by "dissecting" the infection dynamics into "susceptibility" $\rightarrow$ "infection" $\rightarrow$ "severity of infection" $\rightarrow$ "outcome of infection with corresponding financial consequences". Policies can be defined for diseases with or without wave-like dynamics, taking into account the real-time state of disease spread. Furthermore, various levels of infection severity can be incorporated, resulting in different levels of healthcare costs. It should be noted that this temporal dynamics can be viewed as a monitoring mechanism for assessing the "fair" cost of coverage to the insurance company and the insured. Otherwise, it could be viewed as a weekly adjustment mechanism that may be unrealistic due to high premium adjustment costs. Nonetheless, this mechanism guarantees full policy solvency for the insurer. The model also considers the variability in potential benefits derived from the policy, observing that for each outcome of infection (except mortality), there is a specific volatility in the length of hospitalization, thus introducing volatility not only in the outcome of the infection itself but also in the duration of the treatment plan once a certain outcome occurs.

Of course, given the complexity of the topic, there are several potential extensions and avenues for further development of the paper, as well as implications for academic literature in this field, although those are not directly addressed in the paper. From the portfolio management perspective, regarding the observation that assumptions should be weakened to enhance practitioner interest: it's noteworthy that insurers typically adopt a portfolio perspective, even in individual insurance, in order to compute, in an effective and reliable way, the technical reserves. This suggests that assumptions could be revised to reflect this perspective, potentially making the paper more relevant and engaging for practitioners. Exploring how such adjustments could be made without compromising the integrity of the analysis could offer valuable insights and broaden the applicability of the research findings. From the normative perspective, it would be interesting to discuss Solvency 2 aspects raising important considerations about the usability of the model for calculating premium risk. Discussing potential adaptations of the model to better capture pandemic premium risk could open up new avenues for research and practical application. This could involve exploring adjustments or additional components to the model that would enable insurers to assess and manage premium risk in pandemic scenarios more accurately.

Naturally, in this model, reinfection and infection outcome transfers are considered, taking into account the possibility that if an individual has not experienced mortality, the statistical unit in question can fall back into the susceptible state and then become infected again, considering data from subsequent weeks. The same reasoning applies to a statistical unit that is in intensive care in one observation week and then moves to the hospitalization outcome in the next one. It should be noted that an infected person remains immune for approximately six months, but the model can be further extended based on the nature of the disease. Therefore, we present a versatile model capable of fitting the dynamics of the disease well, considering the possibility and timing of infection, as well as the potential types of disease consequences.

7 Declarations

Conflict of interest/Competing interests: The Authors have declared no conflict of interest.

Funding: No funding are provided for this research

Availability of data and materials: Data and materials available on *OSF.io* storage at `osf.io/vw6nu/`

Code availability: Code script available on *OSF.io* storage at `osf.io/vw6nu/`

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