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The usefulness of Multi-criteria sorting methods: a case study in the automotive sector

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A variety of Multi-criteria Decision Aiding methods (MCDA) have been proposed in the literature and their applications are increasingly wide spreading in several sectors. However, the use of such methods is very limited and rarely considered in manufacturing companies. The aim of this paper is to understand how useful MCDA methods are and how they can actually contribute to the performance improvement of manufacturing processes. More in detail, we aim to understand the practical impact of MCDA methods and their shortcomings when applied to classify manufacturing anomalies in automotive companies. In this sense, we compare the use of two sorting MCDA methods, the AHPSort and the ELECTRE TRI method with the procedure adopted by an automotive manufacturing company to sort manufacturing anomalies in one of the biggest plants in the South of Italy. We show that, despite the methods requiring an interactive process and the involvement of the decision maker, the procedure was well accepted by the management of the plant and helped them to reflect on how the classification of the anomalies was conducted.

Keywords: MCDA, AHPSortII, ELECTRE TRI, AUTOMOTIVE SECTOR

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1 Introduction

Multi-criteria Decision Aiding methods (MCDA) support Decision Makers (DMs), helping them to better frame the problem, the alternatives and the criteria to adopt in order to reach the pre-established goals (Figueira et al., 2005). While in several domains the use of MCDA methods (Ishizaka and Nemery, 2013) is becoming increasingly more popular, their adoption is still very limited in manufacturing industries (Bernroider and Schmöllerl, 2013). Indeed, examples have been proposed (i.e. Uz Zaman et al. 2018) but still there is a lack of awareness of their usefulness. The main drawbacks of adopting MCDA methods in real world context can be summarised as:

- different MCDA methods can lead to different results (Ishizaka and Nemery, 2013);
- methods are difficult to understand (Ishizaka and Nemery, 2013);
- lack of reliable software or easiness of their use (Ishizaka et al., 2012);
- the level of interaction with DMs can generate an overwhelming cognitive burden (Miettinen et al., 2008);
- the management may struggle to understand what type of methods is needed (Cinelli et al., 2020).

Among the strategies identified in the literature to advance the use of MCDA methods in real-life studies, (Cinelli et al., 2020) suggested that the practical impact of MCDA methods and their shortcomings should be experimented in real life contexts. In this sense, we aim to understand how MCDA methods can help in supporting decisions in a very complex and competitive sector such as the automotive manufacturing sector. In particular, we test the usefulness of two MCDA methods in a comparative study (Ishizaka and Siraj, 2018). Let us recall that, generally, MCDA methods are implemented with several aims, such as: selecting the best alternative or reduce the group of alternatives to be analysed (e.g., Saaty 1980); ranking the alternatives with descending preference (e.g., Larichev 2001); describing the consequences of implementing one or a group of alternatives (e.g., Brans and Mareschal 1994) or finally sorting the alternatives into pre-defined ordered classes (e.g., Doumpos and Figueira 2019). In this sense, several sorting MCDA methods have been proposed that suggest how to sort the alternatives in classes ordered from the most preferred to the least preferred (Figueira et al., 2013). Among them we can find the MCDA sorting methods based on the concept of full aggregation. In these procedures an evaluation is provided for each criterion and then, the evaluations of the different criteria are aggregated into a score according to different principles (examples of these methods are UTADISGMS (Greco et al., 2010), MACBETHSort (Ishizaka and Gordon, 2017) AHPSortII (Ishizaka et al., 2020). Alternatively, we have methods based on defining an outranking relation among the alternatives ELECTRE TRI (de Miranda Mota and de Almeida, 2012), PROMETHEE SORT (Sarrazin et al., 2018), FlowSort (Pelissari et al., 2019) and ELECTRE TRI-NC (Madhooshiarzanagh and Abi-Zeid, 2021). In this paper we aim to understand if sorting MCDA methods can

help the management of the quality control process of an automotive company. More in detail, we aim to:

- compare the use of MCDA methods with the procedure adopted by the company;
- confront the differences from the DM's point of view in adopting a full aggregation approach in comparison with an outranking approach;
- reflect on how the initial parameters needed for the MCDA methods influence the understanding and the willingness of the management to adopt them;
- comprehend how the presentation of the method from the analyst determine the judgment on the method itself by the management of the company.

To achieve those aims we conducted an analysis of the manufacturing anomalies of an automotive company in Italy. In particular, we selected two of the most adopted MCDA sorting methods (AHPSortII, Ishizaka et al., 2020 and \rightarrow ELECTRE TRI, Alvarez et al., 2021) and we applied them to classify manufacturing anomalies in an automotive plant based in Italy. We also compared the results obtained by our methods with the ones provided by the methodology adopted by the company. We show that MCDA sorting methods simplify the process of sorting alternatives by using a more reliable approach. The paper is organised as follows. In the next section we review the literature about comparative analysis of MCDA methods. In Section 3 we briefly recall the description of the two MCDA sorting methods. In Section 4 we introduce the case study while in Section 5 we describe the results obtained by the selected MCDA methods. Finally, Section 6 concludes the paper.

2 Literature review

The problem of choosing a MCDA methods is historically well-known in the literature on MCDA methods (Ozernoy, 1987) and different suggestions have been provided on how the selection should be conducted. Recently, the most comprehensive approach has been suggested by Cinelli et al. (2020) where the authors proposed a complete taxonomy that may help the DMs and the analyst to select the most appropriate approach. Differently from the other papers, they focus on the whole set of characteristics that a MCDA method should have according to several elements such the type of problem that needs to be solved, the participation of the stakeholders, the mechanism to elicit preferences and so on. The other very popular strand of research, concerns the application of two or more MCDA methods for handling the same problem. This approach serves to help the DM in understanding the impact of the alternatives on certain criteria and therefore on the overall usefulness of a particular method (Triantaphyllou, 2000). In this sense, Opricovic and Tzeng (2004) carried out a comparative study between two MCDA methods (VIKOR, Opricovic and Tzeng, 2007 and TOPSIS, Pavić and Novoselac, 2013) to analyse the strengths and the weaknesses of each method adopting a numerical example describing the selection of a mountain climber of a particular destination. A more interesting approach is comparing different MCDA methods in a real life applications. In this

regard, several applications have been provided. For example, Mela et al. (2012) compared five different methods (the weighted sum method, the weighted product method, VIKOR, TOPSIS, PROMETHEE II, and a procedure based on the PEG-theorem) to handle a building design problem. Similarly, but in the energy field, Lee and Chang (2018) carried out a comparative study of four MCDA methods (weighted sum method, VIKOR, TOPSIS and \rightarrow ELECTRE) to classify renewable energy sources for electricity generation. Likewise, Kolios et al. (2016) adopted the TOPSIS method and the PROMETHEE method to envisage the optimal design alternative for wind turbines. In other fields comparative studies were conducted by Mulliner et al. (2016), with the purpose of assessing the housing affordability, by Karande et al. (2016) to rank different types of industrial robot selection and by Stanujkic et al. (2013) to analyze the different classifications of banks obtained with the use of different MCDA methods. Comparative case studies have been conducted also in the automotive sector. For example, Moradian et al. (2019) has implemented three different MCDA methods to select the most appropriate model for a particular component, while Ramkumar et al. (2009) adopted AHP and TOPSIS for the ranking of third part logistic providers. While all these applications were made on the basis of a specific real case study that need to be handled, Ishizaka and Siraj (2018) conducted an ad-hoc designed experiment adopting incentive mechanisms with the aim of verifying whether MCDA methods really helped DMs and which method among AHP (Saaty, 1980), SMART (Risawandi and Rahim, 2016) and MACBETH (e Costa et al., 2016) was considered the most useful. They found that MCDA methods helped participants in their decision-making process by providing indications on alternatives they ignored in their initial preferences and that the use of AHP and SMART as decision support tools was retained particularly useful.

3 Two sorting MCDA methods

Let us consider a set of anomalies $A = \{a_1, \dots, a_K\}$, identified in a specific period, to be evaluated using a set of criteria $G = \{g_1, \dots, g_J\}$, with $g_j(a_k)$ representing the evaluation of anomaly a_k on criterion g_j . Our aim is to assign each anomaly a_k to a set of ordered classes $C = \{C_1, C_i, \dots, C_I\}$. We define for each class $C_i \in C$ and for each criterion $g_j \in G$:

- A set of central profiles $CP_{ij} = \{cp_{1j}, \dots, cp_{ij}, \dots, cp_{Ij}\}$;
- A set of limiting profiles $LP_{ij} = \{lp_{1j}, \dots, lp_{ij}, \dots, lp_{Ij}\}$.

We also assume to have a weight w_j defined for each criterion $g_j \in G$. Several methods can be used to determine the set of weights Abastante et al. (2020) as for example the eigenvalue method (Ishizaka et al., 2020). In the following we briefly describe the two MCDA sorting methods adopted in this study.

3.1 AHPSortII

The AHPSortII (Ishizaka et al., 2020) is a sorting method used when dealing with a large number of alternatives. In order to reduce the number of pairwise comparisons

conducted between the alternatives, this method introduced, for each criterion g_j , the representative profiles s_{oj} with $o = \{1, \dots, rp_j\}$ that are well distributed points in the scale of each criterion and that eventually can correspond to the central profiles CP_{ij} or to limiting profiles LP_{ij} as defined earlier. The analyst builds with the DM the pairwise comparison matrix for each criterion g_j and for each the representative profiles s_{oj} . Then, thanks to the eigenvalue method (Saaty, 2003) the local priorities p_{oj} for each representative profile s_{oj} are identified. Following the description introduced in Ishizaka et al. (2020), the local priority p_{kj} of the anomaly a_k according to criterion g_j can be found thanks to the use of linear interpolation, as follows:

$$p_{kj} = p_{oj} + \frac{(p_{o+1j} - p_{oj})}{(s_{o+1j} - s_{oj})} \times (g_j(a_k) - s_{oj}) \quad (1)$$

Finally, the determination of the global priority according to all the criteria $g_i \in G$ of anomaly $a_k \in A$ is obtained aggregating the weighted local priorities as

$$p_k = \sum_{j=1}^{\Gamma} p_{kj} \times w_j$$

for each anomaly a_k .

Analogously, the global priority for the central profiles is defined as

$$p_{cp_i} = \sum_{j=1}^{\Gamma} p_{ij} \times w_j$$

and for the limiting profiles as

$$pl_{p_i} = \sum_{j=1}^{\Gamma} p_{ij} \times w_j.$$

Lastly, to assign each anomaly a_k to a class C_i the following two procedures can be adopted:

- If central profiles CP_{ij} have been defined, the anomaly a_k are sorted according to the closeness to the representative central profile c_{pi} of class C_i in terms of their global priorities (Ishizaka et al., 2020).
- If limiting profiles have been defined, the anomaly a_k is assigned to the class C_i which has an lp_i just below the global priority p_k (Ishizaka and Siraj, 2018).

3.2 →ELECTRE TRI

→ELECTRE TRI (Mousseau et al., 2000) is a sorting method based on the use of an outranking relations such as an anomaly a_1 is “at least as good as” a different anomaly a_2 (Fattoruso et al., 2019). Let us point out that in our case being “good” means that

the anomaly is more worrying or serious for a particular criterion $g_j \in G$. The method is constructed in two steps. Following the description introduced by Bouyssou and Marchant (2015), in the first step an outranking relation s is defined for each anomaly $a_1, a_2 \in A$ as:

$$s(a_1, a_2) = \sum_{j=1}^{\Gamma} w_j c_j(a_1, a_2)$$

Where:

- a_1 and a_2 are two generic anomalies $\in A$;
- w_j is a non-negative weight assigned for each criterion g_j ;
- $c_j(a_1, a_2)$ is partial concordance relation, i.e. assuming p_j as a non-negative preference threshold required for each criterion $g_j \in J$, we have:
 - $c_j(a_1, a_2) = 1$ if $g_j(a_2) - g_j(a_1) \leq p_j$;
 - $c_j(a_1, a_2) = 0$ if $g_j(a_2) - g_j(a_1) > p_j$.

Then, once identified a cutting level $\lambda \in [0, 1]$, a binary relation S_λ on two anomalies a_1 and a_2 can be defined as:

$$a_1 S_\lambda a_2 \Leftrightarrow s(a_1, a_2) \geq \lambda,$$

i.e. anomaly a_1 is at least as good as anomaly a_2 . Then, between pair of anomalies we can identify the following three situations:

- if $a_1 S_\lambda a_2$ and Not $a_2 S_\lambda a_1$, then anomaly a_1 is strictly preferred to anomaly a_2 ;
- if $a_1 S_\lambda a_2$ and $a_2 S_\lambda a_1$, then anomaly a_1 is indifferent to anomaly a_2 ;
- if Not $a_1 S_\lambda a_2$ and Not $a_2 S_\lambda a_1$ then anomaly a_1 is incomparable to anomaly a_2 .

The second step is exploitation of the outranking relation S_λ in order to assign each alternative to a specific ordered classes C_i . For this aim, it is possible distinguish between the \rightarrow ELECTRE TRI-B method and the \rightarrow ELECTRE TRI-C (Bouyssou and Marchant, 2015).

More in detail, the \rightarrow ELECTRE TRI-B used for sorting anomalies a_k in class C_i the limiting profiles LP_{ij} where the lower limiting profile of C_i is lp_{ij} and the upper limiting profile of C_i is lp_{i+1j} . Then, two different strategies can be adopted:

- \rightarrow ELECTRE TRI-B-pc an anomaly a_k is assigned to the class C_i if, according to the relation S_λ , a_k is at least good as the lower limiting profile lp_{ij} and is not at least as good as its upper limiting profile lp_{i+1j}
- \rightarrow ELECTRE TRI-B-pd an anomaly a_k is assigned to the class C_i if, according to the relation S_λ , the upper limiting profile lp_{i+1j} is better than a_k and the lower limiting profile lp_{ij} is not better than a_k .

The \rightarrow ELECTRE TRI-C method adopts central profiles CP_{ij} to assign an anomaly a_k to a class C_i . Let us define for each $a_1, a_2 \in A$ a selecting function $\rho(a_1, a_2) = \min(s(a_1, a_2), s(a_2, a_1))$. In this case, two different components should be adopted:

- \rightarrow ELECTRE TRI-C-d
 - Starting from the last central profile cp_{Ij} until $s(a_k, cp_{ji}) \geq \lambda$,
 - for $i = I$, assign a_k to C_I ,
 - for $1 \geq i \geq r - 1$, assign a_k to C_i if $\rho(a_k, cp_{ij}) \geq \rho(a_k, cp_{i+1j})$, otherwise assign a_k to C_{i+1} ,
 - for $i = 0$, assign a to C_1 .
- \rightarrow ELECTRE TRI-C-a
 - Starting from the first central profile cp_{1j} until the first value k such that $s(cp_{kj}, a) \geq \lambda$,
 - for $i = 1$, assign a_k to C_1 ,
 - for $2 \leq i \leq I$, assign a_k to C_k if $\rho(a_k, cp_{kj}) \geq \rho(a_k, cp_{k-1j})$, otherwise assign a_k to C_{i-1} ,
 - for $k = I + 1$, assign a_k to C_I .

Let us highlight that the both the components \rightarrow ELECTRE TRI-C-d and \rightarrow ELECTRE TRI-C-a should be adopted conjointly, meaning that in some cases the assignment of an anomaly a_k could be between two or more classes (Figueira et al., 2013). Let us also highlight that veto thresholds and indifference thresholds could also be specified. The reader interested in more detail can refer to the paper of Roy et al. (2014).

4 Case study: classifying anomalies in an automotive company

We deal with the classification of the anomalies for an important company in the automotive sector. The company is located in the South of Italy and deals with engine assembly. Their aim is to prevent anomalies from appearing during the production process. In order to minimise anomalies, when eventually these happen, a process is activated to identify and resolve the anomalies manifested as soon as possible. The process constitutes of two steps. In the first step a priority index α is computed for each anomaly $a_i \in A$ as $\alpha_i = \prod_{j=1}^{\Gamma} g_j$. More in detail, the criteria used for the construction of the index are:

- g_1 , i.e. the frequency of the anomalies identify in a specific period (real data detected in the plant);
- g_2 , i.e. the cost in terms of material and hours required to solve the anomalies (a qualitative scale ranging from 1 to 5 defined by the management of the company);

- g_3 , i.e. the severity in terms of the impact of the anomalies on the customers (a qualitative scale ranging from 1 to 5 defined by the management of the company);
- g_4 , i.e. the detection point of the process in which the anomalies is detected (a score from 1 to 25 in based on the point in which the anomalies are identified in the process defined by the management of the company).

The priority index defines a classification of anomalies from the most serious ones, mostly related to the safety or dissatisfaction of the end customer, to the marginal ones, related to productivity and the reduction of the number of defects within the plant. Then, the anomalies can be assigned to a certain number of classes C_i where:

- C_1 , containing the High seriousness anomalies;
- C_2 , containing the Medium seriousness anomalies;
- C_3 , containing the Low seriousness anomalies, and so on.

Specifically, the company classifies the anomalies according to a ranking build on the priority index above defined. For example the first top 50 of anomalies can be assigned to class C_1 , then the top 20 of anomalies is assigned to class C_2 and so on.

From the scope of our study, We consider a set of 38 anomalies $A = \{a_1, \dots, a_{38}\}$ and four criteria $G = (g_1, \dots, g_4)$, detected into the company between June 2020 and March 2021. Considering each criterion $g_j \in G$, we reported in Table 1 the evaluations $g_j(a_k)$ for each anomaly $a_k \in A$. We also report for each anomaly $a_k \in A$ the class C_i to which it belongs according to a classification introduced by the company, indicated with \rightarrow CMP that distributes the anomalies in three classes.

From the analysis of the current methodology and from the dialogue with the management of the company, three main weaknesses emerged. First, it appears that the criteria have different importance with severity being the most important one, however in the priority index defined by the company all the criteria assume the same importance. Instead, we believe that a methodology that takes into account this aspect should be adopted. In this sense in the literature there are various methods for determining the weights, i.e. the importance of the different criteria, and among the most intuitive and adopted we can recall the SRF method (Abastante et al., 2020) or the AHP method (Saaty, 2003). Second, the priority index is only based on the multiplication of the evaluation of the criteria while, as explained earlier, MCDA methods are based on more theoretically funded principles (Ishizaka and Nemery, 2013). Third, the construction of priority classes is based only on an assignment based on the quantity of anomalies that have happened instead of being based on a more structured relation among the criteria (Zopounidis and Doumpos, 2002).

In this sense, we believe that the choice of adopting MCDA sorting methodologies can overtake these shortcomings and also be useful for repetitive and/or automatic use.

Table 1: Classification \rightarrow CMP of anomalies $a_k \in A$ according to the company methodology

	g_1	g_2	g_3	g_4	C_i		g_1	g_2	g_3	g_4	C_i
a_1	8	2	25	3	C_1	a_{20}	1	2	10	3	C_2
a_2	1	5	25	5	C_1	a_{21}	1	2	10	3	C_2
a_3	16	1	12	3	C_1	a_{22}	1	2	10	3	C_2
a_4	1	5	25	4	C_1	a_{23}	1	2	10	3	C_2
a_5	1	5	25	4	C_1	a_{24}	2	1	10	3	C_2
a_6	1	5	15	5	C_1	a_{25}	9	1	2	3	C_2
a_7	5	1	12	5	C_1	a_{26}	1	1	10	3	C_2
a_8	7	1	10	3	C_1	a_{27}	1	1	10	3	C_2
a_9	1	1	25	5	C_1	a_{28}	1	1	10	3	C_2
a_{10}	1	1	25	5	C_1	a_{29}	1	1	10	3	C_2
a_{11}	2	2	10	3	C_1	a_{30}	1	1	10	3	C_2
a_{12}	1	3	10	3	C_1	a_{31}	1	1	10	3	C_2
a_{13}	1	1	25	3	C_1	a_{32}	1	1	10	3	C_2
a_{14}	1	1	25	3	C_1	a_{33}	1	2	1	4	C_2
a_{15}	1	2	10	3	C_1	a_{34}	1	1	1	3	C_2
a_{16}	1	2	10	3	C_1	a_{35}	1	1	1	2	C_2
a_{17}	1	2	10	3	C_1	a_{36}	1	1	1	2	C_3
a_{18}	1	2	10	3	C_1	a_{37}	1	1	1	2	C_3
a_{19}	1	2	10	3	C_1	a_{38}	1	1	1	2	C_3

5 Classification of anomalies in classes with the two MCDA sorting methods

To implement the AHPSort II and \rightarrow ELECTRE TRI method we interacted with the manager of the quality process of the plant, who is the DM of the company that dealing with the analysis of the anomalies and subsequent interventions to rectify the processes. First, we asked the DM if he is happy to keep considering the three ordered classes as defined above, i.e. C_1 being the class with the most serious anomalies and C_3 the class with the less serious anomalies. Second, we asked the DM to define the central profiles CP_{ij} (shown in Table 2) and the limiting profiles LP_{ij} (shown in Table 3). Let us point out that, in this phase, we interacted with him thanks to the use of semi-structured questionnaire in order to facilitate the interaction (da Silva Neves and Camanho, 2015).

Third, for the application of both methods we asked the DM to pairwise compare the

Table 2: Central profiles cp_{ij} for each class $C_i \in C$ and for each criterion $g_j \in G$

	g_1	g_2	g_3	g_4
cp_{1j}	16	5	10	5
cp_{2j}	10	2	2	3
cp_{3j}	1	1	1	1

Table 3: Limiting profiles lp_{ij} for each class $C_i \in C$ and for each criterion $g_j \in G$

	g_1	g_2	g_3	g_4
lp_{1j}	10	4	10	4
lp_{2j}	5	2	5	3

four criteria and, thanks to the use of the eigenvalue method (Saaty, 2003), we calculated the weights w_j for each criterion g_j , as shown in Table 4.

Table 4: Weights w_j for each criterion $g_j \in G$

	g_1	g_2	g_3	g_4	w_j
g_1	1	1/2	1/3	1/9	0,049
g_2		1	1/2	1/9	0,077
g_3			1	1/9	0,135
g_4				1/9	0,739

5.1 The obtained classification with the AHPSort II

For sorting the anomalies a_k in classes $C_i \in C$ the AHPSort II identify the so called representative profiles s_{oj} , renamed for simplicity $RP1, \dots, RP6$, and shown in Table 5. We identified these representative profiles, i.e. well represented points on the scale of each criterion, following the procedure adopted in Ishizaka et al. (2020).

Then, we asked the DM to pairwise compare the representative profiles s_{oj} with the central profiles cp_{ij} and with the limiting profiles lp_{ij} , respectively. Again, thanks to the use of the eigenvalue method (Saaty, 2003) the local priorities p_{oj} for each representative profile s_{oj} can be identified. Finally, the local priority p_{kj} and the global priority p_k for each anomaly a_k can be identified. In Table 6 we report, as an example, the local priority p_{kj} and global priority p_k calculated with respect to the central Profiles cp_{ij} .

Table 5: Representative profiles s_{oj} for the application of the AHPSort II

	g_1	g_2	g_3	g_4
<i>RP1</i>	0	0	0	0
<i>RP2</i>	3,2	1	5	1
<i>RP3</i>	6,4	2	10	2
<i>RP4</i>	9,6	3	15	3
<i>RP5</i>	12,8	4	20	4
<i>RP6</i>	16	5	25	5

Table 6: Local and Global Priorities for anomalies $a_k \in A$ with respect to the central profiles cp_{ij}

	p_{k1}	p_{k2}	p_{k3}	p_{k4}	p_k		p_{k1}	p_{k2}	p_{k3}	p_{k4}	p_k
a_1	0,097	0,035	0,426	0,055	0,106	a_{20}	0,012	0,035	0,046	0,055	0,050
a_2	0,012	0,303	0,266	0,308	0,287	a_{21}	0,012	0,035	0,046	0,055	0,050
a_3	0,415	0,013	0,107	0,055	0,076	a_{22}	0,012	0,035	0,046	0,055	0,050
a_4	0,012	0,303	0,426	0,166	0,204	a_{23}	0,012	0,035	0,046	0,055	0,050
a_5	0,012	0,303	0,426	0,166	0,204	a_{24}	0,015	0,013	0,046	0,055	0,049
a_6	0,012	0,303	0,144	0,308	0,271	a_{25}	0,127	0,013	0,029	0,055	0,052
a_7	0,035	0,013	0,107	0,308	0,245	a_{26}	0,012	0,013	0,046	0,055	0,048
a_8	0,066	0,013	0,046	0,055	0,051	a_{27}	0,012	0,013	0,046	0,055	0,048
a_9	0,012	0,013	0,426	0,308	0,287	a_{28}	0,012	0,013	0,046	0,055	0,048
a_{10}	0,012	0,013	0,426	0,308	0,287	a_{29}	0,012	0,013	0,046	0,055	0,048
a_{11}	0,015	0,035	0,046	0,055	0,050	a_{30}	0,012	0,013	0,046	0,055	0,048
a_{12}	0,012	0,097	0,046	0,055	0,055	a_{31}	0,012	0,013	0,046	0,055	0,048
a_{13}	0,012	0,013	0,426	0,055	0,100	a_{32}	0,012	0,013	0,046	0,055	0,048
a_{14}	0,012	0,013	0,426	0,055	0,100	a_{33}	0,012	0,035	0,012	0,166	0,128
a_{15}	0,012	0,035	0,046	0,055	0,050	a_{34}	0,012	0,013	0,012	0,055	0,044
a_{16}	0,012	0,035	0,046	0,055	0,050	a_{35}	0,012	0,013	0,012	0,032	0,027
a_{17}	0,012	0,035	0,046	0,055	0,050	a_{36}	0,012	0,013	0,012	0,032	0,027
a_{18}	0,012	0,035	0,046	0,055	0,050	a_{37}	0,012	0,013	0,012	0,032	0,027
a_{19}	0,012	0,035	0,046	0,055	0,050	a_{38}	0,012	0,013	0,012	0,032	0,027

Following that, according to the procedure defined in Subsection 3.1 the anomaly are assigned to each class. In particular we call, the classification obtained adopting the central profiles cp_{ij} as $AHPS_{CP}$ and the classification obtained adopting the limiting profiles lp_{ij} as $AHPS_{LP}$. Those classifications are reported in Table 7.

Table 7: Classification of anomalies $a_k \in A$ with AHPSORT II

	$AHPS_{CP}$	$AHPS_{LP}$		$AHPS_{CP}$	$AHPS_{LP}$
a_1	C_2	C_2	a_{20}	C_2	C_2
a_2	C_1	C_1	a_{21}	C_2	C_2
a_3	C_2	C_2	a_{22}	C_2	C_2
a_4	C_1	C_1	a_{23}	C_2	C_2
a_5	C_1	C_1	a_{24}	C_2	C_2
a_6	C_1	C_1	a_{25}	C_2	C_2
a_7	C_1	C_1	a_{26}	C_2	C_2
a_8	C_2	C_2	a_{27}	C_2	C_2
a_9	C_1	C_1	a_{28}	C_2	C_2
a_{10}	C_1	C_1	a_{29}	C_2	C_2
a_{11}	C_2	C_2	a_{30}	C_2	C_2
a_{12}	C_2	C_2	a_{31}	C_2	C_2
a_{13}	C_2	C_2	a_{32}	C_2	C_2
a_{14}	C_2	C_2	a_{33}	C_2	C_1
a_{15}	C_2	C_2	a_{34}	C_2	C_3
a_{16}	C_2	C_2	a_{35}	C_3	C_3
a_{17}	C_2	C_2	a_{36}	C_3	C_3
a_{18}	C_2	C_2	a_{37}	C_3	C_3
a_{19}	C_2	C_2	a_{38}	C_3	C_3

In Table 7 it is possible observe that the sorting of anomalies in Classes C_i , with AHPSortII is very similar with the use of central profiles cp_{ij} or with the use of limiting profiles lp_{ij} .

5.2 The obtained classification with the \rightarrow ELECTRE-TRI

To implement the \rightarrow ELECTRE-TRI-C and the \rightarrow ELECTRE-TRI-B we need to define the preference, the indifference and the veto thresholds. According to Roy et al. (2014) the analyst should interact with the DM to define those parameters. The DM specified that, even in case of qualitative scale, the difference between each level was significative and that, from the company point of view, it was not necessary to exploit situations

in which uncertainty of the data was required at least at this stage. Therefore, for each criterion $g_i \in G$ the preference thresholds, the indifference thresholds and the veto thresholds were supposed to be equal to zero.

Then, the analyst employing the software MCDA ULAVAL¹ conducted the analysis adopting both the limiting profiles and the central profiles. In Table 8 we report the anomalies a_k sorted in Classes C_i according to the four components of the \rightarrow ELECTRE method as explained in Subsection 3.2, i.e.:

- \rightarrow ELECTRE TRI-B-pc (abbreviation \rightarrow TRI-B-pc), with the limiting profiles lp_{ij} ;
- \rightarrow ELECTRE TRI-B-pd (abbreviation \rightarrow TRI-B-pd), with the limiting profiles lp_{ij} ;
- \rightarrow ELECTRE TRI-C-d (abbreviation \rightarrow TRI-C-d), with the central profiles cp_{ij} ;
- \rightarrow ELECTRE TRI-C-a (abbreviation \rightarrow TRI-C-a), with the central profiles cp_{ij} .

5.3 Discussion

After the assignment of each anomaly $a_k \in A$ to a class $C_i \in C$ with the use of the two MCDA sorting methods, we showed the results to the DM. To facilitate the understanding of the different classifications, we illustrate them as reported in Table 9, i.e. comparing the classification obtained by the company \rightarrow CMP with the classifications obtained by the MCDA methods. More in detail, the symbol = means that the considered MCDA method assigned anomaly a_k to the same class, the symbol \uparrow means that the considered MCDA method assigned anomaly a_k to a class with higher seriousness while the symbol \downarrow assigned anomaly a_k to a lower seriousness class.

It is interesting to note that the classifications obtained with the AHPSortII methods are quite robust with very little changes of classes obtained. Instead, for \rightarrow ELECTRE methods many more variations are presented, especially when the adopting the \rightarrow ELECTRE TRI-B-pd and the \rightarrow ELECTRE TRI-C-a. Let us remind that while for the \rightarrow ELECTRE TRI-B those classifications can be considered as two different classifications, for the \rightarrow ELECTRE TRI-C the anomaly should be considered as between the classes obtained with \rightarrow ELECTRE TRI-C-D and \rightarrow ELECTRE TRI-C-A Bouyssou and Marchant (2015).

The DM found both the proposed methodologies interesting to address the problem of sorting anomalies in ordered classes. He also provided some more specific comments about the different methodologies. First, he was more comfortable with the AHPSortII method in comparison to the \rightarrow ELECTRE methods. Indeed, the full aggregation approach is easier to understand in the business context because the identification of final global scores allows to directly compare the anomalies. Instead, he found that the outranking approach based on the principle “an anomaly is at least as bad as another” is more difficult to understand because it is very different from the methodological approach of the company. He particularly struggled with the understanding of the incomparability concept between anomalies.

¹Available at <http://cersvr1.fsa.ulaval.ca/mcda/>.

Second, the DM expressed his opinion on the initial parameters needed for the MCDA methods. According to him, the identification of the central profiles cp_{ij} is simpler than the identification of the limiting profiles lp_{ij} . Indeed, once the number of classes has been identified the DM is more able to identify a central profile for each class than the limiting profiles that separate a class from another.

Third, the DM found very useful to weigh the criteria with the method of pairwise comparisons (Cavallo et al., 2019) and recognise that weighting the criteria was an important step to define the classification of the anomalies.

Finally, the DM pointed out the different characteristics that he noted for each classification, i.e.:

- The classifications $AHPS_{CP}$ and $AHPS_{LP}$ the DM did not find substantial changes in the attribution of anomalies in classes using the central profiles cp_{ij} or the limiting profiles lp_{ij} . In this sense, the DM has oriented his preference in using the $AHPSORTIII_{CP}$ because of the anomaly a_{33} assigned to a more serious class.
- The DM expressed a preference for the classification \rightarrow TRI-B-PD in comparison with the classification \rightarrow TRI-B-PC because more anomalies were assigned to the C_1 class, with the most serious anomalies. For the DM it is essential to assign anomalies in all the identified classes and to identify the most serious anomalies to be solved.
- the DM expressed some doubts on the usefulness of the classification \rightarrow TRI-C-D because there were no anomalies assigned to class C_3 . However, the analyst explained that this needed to be considered as a part of the classification together with the one obtained with \rightarrow TRI-C-A. He was happy to consider that he could have some flexibility in assigning the anomalies to the classes.

6 Conclusion

In this paper we applied two sorting MCDA methodologies to classify the anomalies happening in a manufacturing process for an automotive company. The implementation highlighted that the use of these methods can help and support DMs in improving the quality control process of such a complex environment. The process was smoothly run but it must be noted that the relationship with the DM is essential to the success of the implementation. It is also noteworthy observing that the differences in classification obtained, pushed the DM to think about the way in which they classify anomalies and to reflect about potential improvements of the systems.

As future developments we believe that some other methodologies could be tried to help the DMs in the quality control process even procedures that require less parameters to be defined by the DM. In addition, it would be important to verify if the the definition of subcriteria (Leal, 2020) or even the analysis of interaction among the criteria could influence the classification obtained (Figueira et al., 2009). The use of fuzzy MCDA methods (Meshram et al., 2019) could be considered in order to give some more flexibility

to the DM to the final assignment (Migdady and Al-Talib, 2018). In future works it would also be interesting to analyze and identify the factors that influence DMs during the choice process (Ibrahim, 2016). Furthermore, the study of the same problem even with a group of experts could be conducted and some MCDA group methodologies could be adopted (Marcarelli and Squillante, 2020).

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Table 8: Classification of anomalies $a_k \in A$ with \rightarrow ELECTRE TRI methods

	\rightarrow TRI-B-pc	\rightarrow TRI-B-pd	\rightarrow TRI-C-d	\rightarrow TRI-C-a
a_1	C_1	C_2	C_1	C_2
a_2	C_1	C_3	C_1	C_2
a_3	C_1	C_3	C_1	C_2
a_4	C_1	C_3	C_1	C_2
a_5	C_1	C_3	C_1	C_2
a_6	C_1	C_3	C_1	C_2
a_7	C_1	C_3	C_1	C_2
a_8	C_2	C_3	C_2	C_2
a_9	C_1	C_3	C_1	C_2
a_{10}	C_1	C_3	C_1	C_2
a_{11}	C_2	C_3	C_2	C_2
a_{12}	C_2	C_3	C_2	C_2
a_{13}	C_1	C_3	C_1	C_2
a_{14}	C_1	C_3	C_1	C_2
a_{15}	C_2	C_3	C_2	C_2
a_{16}	C_2	C_3	C_2	C_2
a_{17}	C_2	C_3	C_2	C_2
a_{18}	C_2	C_3	C_2	C_2
a_{19}	C_2	C_3	C_2	C_2
a_{20}	C_2	C_3	C_2	C_2
a_{21}	C_2	C_3	C_2	C_2
a_{22}	C_2	C_3	C_2	C_2
a_{23}	C_2	C_3	C_2	C_2
a_{24}	C_2	C_3	C_2	C_2
a_{25}	C_2	C_3	C_2	C_3
a_{26}	C_2	C_3	C_2	C_2
a_{27}	C_2	C_3	C_2	C_2
a_{28}	C_2	C_3	C_2	C_2
a_{29}	C_2	C_3	C_2	C_2
a_{30}	C_2	C_3	C_2	C_2
a_{31}	C_2	C_3	C_2	C_2
a_{32}	C_2	C_3	C_2	C_2
a_{33}	C_2	C_3	C_2	C_2
a_{34}	C_3	C_3	C_2	C_3
a_{35}	C_3	C_3	C_2	C_3
a_{36}	C_3	C_3	C_2	C_3
a_{37}	C_3	C_3	C_2	C_3
a_{38}	C_3	C_3	C_2	C_3

Table 9: Classification of anomalies $a_k \in A$ according to the company methodology \rightarrow CMP in comparison with the results obtained by the two MCDA sorting methods

	\rightarrow CMP	$AHPS_{CP}$	$AHPS_{LP}$	\rightarrow TRI-B-pc	\rightarrow TRI-B-pd	\rightarrow TRI-C-d	\rightarrow TRI-C-a
a_1	C_1	\downarrow	\downarrow	$=$	\downarrow	$=$	\downarrow
a_2	C_1	$=$	$=$	$=$	\downarrow	$=$	\downarrow
a_3	C_1	\downarrow	\downarrow	$=$	\downarrow	$=$	\downarrow
a_4	C_1	$=$	$=$	$=$	\downarrow	$=$	\downarrow
a_5	C_1	$=$	$=$	$=$	\downarrow	$=$	\downarrow
a_6	C_1	$=$	$=$	$=$	\downarrow	$=$	\downarrow
a_7	C_1	$=$	$=$	$=$	\downarrow	$=$	\downarrow
a_8	C_1	\downarrow	\downarrow	\downarrow	\downarrow	\downarrow	\downarrow
a_9	C_1	$=$	$=$	$=$	\downarrow	$=$	\downarrow
a_{10}	C_1	$=$	$=$	$=$	\downarrow	$=$	\downarrow
a_{11}	C_1	\downarrow	\downarrow	\downarrow	\downarrow	\downarrow	\downarrow
a_{12}	C_1	\downarrow	\downarrow	\downarrow	\downarrow	\downarrow	\downarrow
a_{13}	C_1	\downarrow	\downarrow	$=$	\downarrow	$=$	\downarrow
a_{14}	C_1	\downarrow	\downarrow	$=$	\downarrow	$=$	\downarrow
a_{15}	C_1	\downarrow	\downarrow	\downarrow	\downarrow	\downarrow	\downarrow
a_{16}	C_1	\downarrow	\downarrow	\downarrow	\downarrow	\downarrow	\downarrow
a_{17}	C_1	\downarrow	\downarrow	\downarrow	\downarrow	\downarrow	\downarrow
a_{18}	C_1	\downarrow	\downarrow	\downarrow	\downarrow	\downarrow	\downarrow
a_{19}	C_1	\downarrow	\downarrow	\downarrow	\downarrow	\downarrow	\downarrow
a_{20}	C_2	$=$	$=$	$=$	\downarrow	$=$	$=$
a_{21}	C_2	$=$	$=$	$=$	\downarrow	$=$	$=$
a_{22}	C_2	$=$	$=$	$=$	\downarrow	$=$	$=$
a_{23}	C_2	$=$	$=$	$=$	\downarrow	$=$	$=$
a_{24}	C_2	$=$	$=$	$=$	\downarrow	$=$	$=$
a_{25}	C_2	$=$	$=$	$=$	\downarrow	$=$	\downarrow
a_{26}	C_2	$=$	$=$	$=$	\downarrow	$=$	$=$
a_{27}	C_2	$=$	$=$	$=$	\downarrow	$=$	$=$
a_{28}	C_2	$=$	$=$	$=$	\downarrow	$=$	$=$
a_{29}	C_2	$=$	$=$	$=$	\downarrow	$=$	$=$
a_{30}	C_2	$=$	$=$	$=$	\downarrow	$=$	$=$
a_{31}	C_2	$=$	$=$	$=$	\downarrow	$=$	$=$
a_{32}	C_2	$=$	$=$	$=$	\downarrow	$=$	$=$
a_{33}	C_2	$=$	\uparrow	$=$	\downarrow	$=$	$=$
a_{34}	C_2	$=$	\downarrow	\downarrow	\downarrow	$=$	\downarrow
a_{35}	C_2	\downarrow	\downarrow	\downarrow	\downarrow	$=$	\downarrow
a_{36}	C_3	$=$	$=$	$=$	$=$	\uparrow	$=$
a_{37}	C_3	$=$	$=$	$=$	$=$	\uparrow	$=$
a_{38}	C_3	$=$	$=$	$=$	$=$	\uparrow	$=$