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What happens when the stock markets are closed?

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The normality of the log-return of stock prices is often assumed by the market players in order to use some useful results, as for instance, the Black-Scholes formula for pricing European options. However, several studies regarding different indexes have shown that the normality assumption of the returns usually fails. In this paper we analyse the normality assumption for intra-day and inter-day log-returns, comparing opening prices and/or closing prices for a large number of companies quoted in the Nasdaq Composite index. We use the Pearson's Chi-Square, Kolmogorov-Smirnov, Anderson-Darling, Shapiro-Wilks and Jarque-Bera goodness-of-fit tests to study the normality assumption. We find that the failure rate in the normality assumption for the log-return of stock prices is not the same for intra-day and inter-day prices, is somewhat test dependent and strongly dependent on some extreme price observations. To the best of our knowledge, this is the first study on the normality assumption for the log-return of stock prices dealing simultaneously with a large number of companies and normality tests, and at the same time considering various scenarios of intra-day, inter-day prices and data trimming.

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1 Introduction

The simplicity of the Black-Scholes formula (see Black and Scholes, 1973) for pricing European options is one of the reasons why the geometric Brownian motion is one of the most popular processes used in mathematical finance. However, for this process is assumed that the log-return of the prices are normal distributed random variables, a condition that many times seems to be not true in practice. In this paper, we test the normality assumption for different formulations of price returns, namely intra-day and inter-day log-returns, for a large set of companies data and using five alternative goodness-of-fit tests. We implement the Pearson's Chi-Square, Kolmogorov-Smirnov, Anderson-Darling, Shapiro-Wilks and Jarque-Bera tests for normality and we consider the log-returns of consecutive closing prices, consecutive closing and opening prices and also opening and closing prices from the same day. We also repeat the tests when we remove some of the higher and lower log-returns from the samples. Importantly, when analyzing cointegration, particularly using Johansen approach in Johansen (1988, 1991), normality is also required. Furthermore, all the above holds for the various financial time series, namely for exchange rate analysis.
The paper is organized as follows: in section 2 we introduce the tests for the less familiarized readers; in section 3 we present and discuss the results of the application of the tests and in section 4 we conclude with some remarks.

2 Goodness-of-fit tests

We apply several goodness-of-fit tests to investigate the normality of the log-returns of stock prices. We choose to use several goodness-of-fit tests from different kinds since all the tests have some advantages and disadvantages. From the "area tests" kind we choose to implement the Pearsons's Chi-square test that compares the real number of observations with the expected number of observations in each class, we can say that compares the data histogram with the histogram of the distribution being tested. Both the Kolmogorov-Smirnov and the Anderson-Darling tests uses the cumulative distribution function and the empirical distribution function and are based in a measure of the discrepancy between those two functions and therefore are considered in the class of "distance tests". Some advantages of this kind of tests is that they are easy to compute, they are more powerful than the Chi-Square test, over a wide range of alternatives, and they provide consistent tests. The Shapiro-Wilks (or Shapiro-Francia) is a test based in the regression between the order statistics of the sample and the mean value of the order statistics from the tested distribution. This test for normality has higher power than the previous ones. Finally, the Jarque-Bera test, is based in the Lagrange multiplier test and computes the sample skewness and kurtosis, to tests if they match with the ones from a normal distribution.

2.1 Pearson's Chi-Square test

The Pearson's Chi-Square test (first introduced in Pearson, 1900), compares the frequency observed in a sample with a particular theoretical distribution, that is, for a number k of classes $C_1, ..., C_k$, mutually exclusive and of total probability, the number of observations O_i in class $i, i = 1, ..., k$ is compared with the expect number of observations E_i, in that same class provided the null hypothesis is true.

The Pearsons's chi-square statistic is,

$$X^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i} \tag{1}$$

having a χ^2 (chi-square) distribution with $k - p - 1$ degrees of freedom, where k is the number of classes and p is the number of estimated parameters. The null hypothesis, of the sample be from a particular distribution, is rejected if the observed value of the statistics is bigger than the critical value obtained from the Chi-square distribution.

2.2 Kolmogorov-Smirnov's test

The Kolmogorov-Smirnov statistic (introduced in Kolmogorov, 1933) allows us to test if a sample of observations is from some completely specified continuous distribution, F_0, by means of comparing a particular kind of distance between the empirical and the theoretical cumulative distribution function.

The test statistic, D_n, is given by,

$$D_n = \max_{1 \le i \le n} \left[\max \left\{ F_0(X_{i:n}) - F_n(X_{i-1:n}), F_n(X_{i:n}) - F_0(X_{i:n}) \right\} \right] \tag{2}$$

where F_0 is the distribution function of the theoretical distribution being tested and F_n is the empirical distribution function. Critical values for this statistic can be found in Birnbaum (1952) or Massey (1951), however, when some parameters of the distribution, considered in the null hypothesis, have to be estimated from the sample, then the commonly tabulated critical points can led to conservative results. In this situation, Lilliefors's critical values (that can be found in Lilliefors, 1967) should be used.

2.3 Anderson-Darling's test

The Anderson-Darling test is the third goodness-of-fitness test that we use to test for normality. More information about this test can be found in Anderson and Darling (1952, 1954), but again, it compares the observed cumulative distribution function to the expected cumulative distribution function as the Kolmogorov-Smirnov test.

The statistic A_n^2, for the Anderson-Darling's test, is defined by,

$$A_n^2 = -n - \frac{1}{n} \sum_{i=1}^{n} (2i - 1) \left[\ln(F_0(X_{i:n})) + \ln(1 - F_0(X_{n-i+1:n})) \right], \tag{3}$$

where, as before, F_0 denotes the distribution function assumed in the null hypothesis. In Anderson and Darling (1954), asymptotic critical points for significance levels of $1\%, 5\%$ and 10% are presented and more extensive tables of critical points obtained from Monte Carlo simulation can be found in Lewis (1961). When the distribution to be tested is normal or exponential and the distribution parameters are unknown and needed to be estimated, we can find the critical values for the Anderson-Darling's statistics in Stephens (1974, 1976).

2.4 Shapiro-Wilk's test

In the Shapiro-Wilk's test, presented in the Shapiro and Wilk (1965), the test statistic W_n was constructed through the regression of the order sample statistics against the expected normal order statistics and is a suitable test when the location and scale parameters are unknown. The test statistic, W_n, is defined by,

$$W_n = \frac{\left(\sum_{i=1}^{n} a_i X_{i:n}\right)^2}{\sum_{i=1}^{n}(X_{i:n} - \bar{X})^2} \tag{4}$$

where

$$\mathbf{a}^T = (a_1, a_2, ..., a_n) = \frac{\mathbf{m}^T \mathbf{V}^{-1}}{\left(\mathbf{m}^T \mathbf{V}^{-1} \mathbf{V}^{-1} \mathbf{m}\right)^{\frac{1}{2}}} \tag{5}$$

with

$$\mathbf{m}^T = (m_1, m_2, ..., m_n), \quad \mathbf{V} = [v_{ij}]_{n \times n} \tag{6}$$

represents the vector of expected values of the standard normal order statistics and the corresponding covariance matrix, respectively. The values for $\mathbf{a}$ and the percentage points of W_n are known up to sample sizes of $n = 50$ and can be found in the original paper. For samples of larger dimension an extension of the Shapiro and Wilk's test can be found in Royston (1982) or in alternative, the Shapiro-Francia statistic (with simpler coefficients and about the same overall power) introduced in Shapiro and Francia (1972) can be used. Percentage points for the Shapiro-Francia statistic can be found in Shapiro and Francia (1972) for sample sizes $n = 35, 50, 51(2)99$ and for samples of larger dimension in Royston (1983). Small values of the statistic are the significant ones, i.e. indicate non-normality.

2.5 Jarque-Bera's test

The last test that we implement is the Jarque-Bera's test, Jarque and Bera (1987). The Lagrange multipliers method was used to derived an asymptotic efficient test where the skewness and kurtosis of the sample data are compared to the ones of the normal distribution.

The test statistic, JB_n, is defined by,

$$JB_n = n\left(\frac{b_1^2}{6} + \frac{(b_2 - 3)^2}{24}\right) \tag{7}$$

where

$$b_1 = \frac{\hat{\mu}_3}{(\hat{\mu}_2)^{\frac{3}{2}}} \text{ and } b_2 = \frac{\hat{\mu}_4}{(\hat{\mu}_2)^2} \quad (8)$$

are the sample estimators of the skewness and kurtosis, with $\hat{\mu}_j, j = 2, 3, 4$ the estimators of the central moments. The large values of the statistic are the significant ones and should be compared with the right tail critical values of a Chi-square distribution with 2 degrees of freedom.

3 Data, testing and discussion

In the following, let SO_i and SC_i be the stock opening and closing prices at day i, respectively. The considered data are the daily prices from years 2005 to 2016 and in each year we only consider the companies from the Nasdaq Composite Index that have more than 220 transactions days and transaction volume of at least 50000 units per day. We represent by K the number of companies selected in each year and from those we take count of how many have their prices failing the normality assumption.

Using the log-returns of closing daily prices, namely,

$$\log\left(\frac{SC_{i+1}}{SC_i}\right) \quad (9)$$

and all the normality tests for a level of significance of 1%, we get the results presented in Table 1.

Table 1: Total (and percentage) of samples rejected by each test for normality at 1% level of significance ($\log(SC_{i+1}/SC_i)$)

Year	K	PCS	KS	AD	SW	JB
2005	901	494(54.83%)	658(73.03%)	763(84.68%)	800(88.80%)	814(90.34%)
2006	1018	546(53.63%)	747(73.34%)	850(83.50%)	909(89.29%)	925(90.86%)
2007	1161	691(59.52%)	923(79.50%)	1050(90.44%)	1085(93.45%)	1095(94.32%)
2008	1180	765(64.83%)	1010(85.60%)	1114(94.41%)	1142(96.78%)	1135(96.19%)
2009	1200	642(53.50%)	934(77.83%)	1090(90.83%)	1105(92.08%)	1104(92.00%)
2010	1260	557(44.21%)	812(64.44%)	1000(79.37%)	1028(81.59%)	1047(83.10%)
2011	1374	686(49.93%)	1007(73.29%)	1208(87.92%)	1267(92.21%)	1287(93.67%)
2012	1399	723(51.68%)	974(69.62%)	1129(80.70%)	1198(85.63%)	1218(87.06%)
2013	1586	989(62.36%)	1253(79.00%)	1400(88.27%)	1475(93.00%)	1486(93.69%)
2014	1441	784(54.41%)	1072(74.39%)	1224(84.94%)	1291(89.60%)	1320(91.60%)
2015	1433	775(54.08%)	1042(72.71%)	1203(83.95%)	1275(88.97%)	1301(90.79%)
2016	1443	902(62.51%)	1182(81.91%)	1318(91.34%)	1361(94.32%)	1379(95.56%)

We can observe that we have normality rejection percentages, ranging from 44% to near 65% with the Pearson's Chi-square test (the one with the smaller rejection rates) to more than 90% if we consider the Jarque-Bera's test (the one with the higher rejection rates). Notice that as expected, the year of 2008 (the year of the financial crisis) is the one with higher normality rejection rate and this for all tests being considered. The year of 2010 is the one with smaller rejection rate and also for all the tests, from 2008 to 2010 we have a decreasing of the normality rejection rate from near 65% to a little more than 44% for the Pearson's Chi-square test and from near 96% to near 83% for the Jarque-Bera's test. For the Kolmogorov-Smirnov's, Anderson-Darling's or Shapiro-Wilk's tests we have decreasing rejection rates from 85.6% to 64.44%, 94.41% to 79.37% and 96.78% to 81.59%, respectively.

Next, we consider closing and opening prices from the same day and the corresponding log-returns,

$$\log\left(\frac{SC_i}{SO_i}\right). \tag{10}$$

For the same level of significance of 1%, as previously, we get the results presented in Table 2.

Table 2: Total (and percentage) of samples rejected by each test for normality at 1% level of significance ($\log(SC_i/SO_i)$)

Year	K	PCS	KS	AD	SW	JB
2005	901	218(24.20%)	411(45.62%)	551(61.15%)	635(70.48%)	672(74.58%)
2006	1018	256(25.15%)	460(45.19%)	607(59.63%)	697(68.47%)	739(72.60%)
2007	1161	449(38.67%)	745(64.17%)	924(79.59%)	980(84.41%)	996(85.79%)
2008	1180	642(54.41%)	909(77.03%)	1058(89.66%)	1095(92.80%)	1096(92.88%)
2009	1200	504(42.00%)	808(67.33%)	980(81.67%)	1023(85.25%)	1028(85.67%)
2010	1260	299(23.73%)	530(42.06%)	727(57.70%)	845(67.06%)	895(71.03%)
2011	1374	471(34.28%)	800(58.22%)	1034(75.25%)	1141(83.04%)	1179(85.81%)
2012	1399	457(32.67%)	752(53.75%)	937(67.00%)	1066(76.20%)	1100(78.63%)
2013	1586	664(41.87%)	1021(64.38%)	1197(75.47%)	1314(82.85%)	1365(86.07%)
2014	1441	510(35.39%)	834(57.88%)	1046(72.59%)	1156(80.22%)	1187(82.37%)
2015	1433	563(39.29%)	902(62.94%)	1123(78.37%)	1220(85.14%)	1255(87.58%)
2016	1443	643(44.56%)	960(66.53%)	1180(81.77%)	1251(86.69%)	1288(89.26%)

The main difference from Table 1 to Table 2 is the overall decreasing of the normality rejection rate in all years and for all considered tests. That decreasing is more notorious in the years of 2005-2006 (with changes as higher as 30% for the Pearson's Chi-square test) and less obvious in the crisis year of 2008 (decreasing at most 10%). Again, the year of 2008 (followed by the 2016 year) is the one with higher percentage of companies with log-returns of stock prices far away from normality and, as before, the year of 2010 is the one with the smaller percentages of normality rejection, decreasing from 54.41% in 2008 to 23.73% for the Pearson's Chi-square test in 2010 and from 92.88% to 71% for the Jarque-Bera's test, the tests with smaller/higher rejection rates, respectively.

Finally, we select the closing prices at day i and opening prices at day $i+1$ and the corresponding log-returns,

$$\log\left(\frac{SO_{i+1}}{SC_i}\right). \tag{11}$$

Again the five normality tests are executed for the same 1% significance level and the results can be found in Table 3.

Table 3: Total (and percentage) of samples rejected by each test for normality at 1% level of significance ($\log(SO_{i+1}/SC_i)$)

Year	K	PCS	KS	AD	SW	JB
2005	901	820(91.01%)	838(93.01%)	851(94.45%)	869(96.45%)	865(96.00%)
2006	1018	955(93.81%)	964(94.70%)	982(96.46%)	999(98.13%)	1000(98.23%)
2007	1161	1098(94.57%)	1132(97.50%)	1149(98.97%)	1153(99.31%)	1156(99.57%)
2008	1180	1147(97.20%)	1172(99.32%)	1179(99.92%)	1179(99.92%)	1178(99.83%)
2009	1200	1012(84.33%)	1116(93.00%)	1164(97.00%)	1168(97.33%)	1168(97.33%)
2010	1260	1027(81.51%)	1121(88.97%)	1190(94.44%)	1229(97.54%)	1236(98.10%)
2011	1374	1248(90.83%)	1344(97.82%)	1370(99.71%)	1368(99.56%)	1367(99.49%)
2012	1399	1243(88.85%)	1329(95.00%)	1360(97.21%)	1362(97.36%)	1370(97.93%)
2013	1586	1459(91.99%)	1526(96.22%)	1549(97.67%)	1556(98.11%)	1553(97.92%)
2014	1441	1340(92.99%)	1382(95.91%)	1408(97.71%)	1422(98.68%)	1423(98.75%)
2015	1433	1342(93.65%)	1396(97.42%)	1418(98.95%)	1422(99.23%)	1421(99.16%)
2016	1443	1393(96.54%)	1425(98.75%)	1438(99.65%)	1442(99.93%)	1441(99.86%)

From the observation of Table 3 is obvious that for this data the results are of almost complete rejection of the normality assumption. In fact, even for the most favorable test in not rejecting the normality assumption (the Pearson's Chi-square test), we have more than 80% of rejection rate, in the most favorable year of 2010.

Although being test depending, the overall result is that the normality assumption is largely rejected for all tests in most years but in higher percentages when we consider prices from one trading day to another (inter-day prices) as we observe in Tables 1 or 3 and lower percentages when we consider the opening and closing prices from the same day (intra-day prices, as in Table 2). This emphasises the idea that the non-normality of the log-returns of the stock prices seems to be more dependent from what happens when the markets are closed (see Table 3).

Since the rejection rates are higher when we apply the Jarque-Bera's test and because this particular test is more sensitive to extremal data it seems reasonable that some of the reason for the normality assumption failure is due to that specific observations. In fact, we observe that if we remove some of the more extreme observations from the log-returns data and we perform the same normality tests we get very different results. Repeating the normality tests for trimmed samples, with the higher five and lower five log-returns removed, we get the results presented in Tables 4, 5 and 6. Notice that, when we remove 10 observations from the sample with approximately 250 entries, we are removing about 4% of the observations.

Table 4: Total (and percentage) of samples rejected by each test for normality at 1% level of significance, when the data is trimmed ($\log(SC_{i+1}/SC_i)$)

Year	K	PCS	KS	AD	SW	JB
2005	901	113(12.54%)	184(20.42%)	187(20.75%)	194(21.53%)	86(9.54%)
2006	1018	110(10.81%)	198(19.45%)	218(21.41%)	216(21.22%)	85(8.35%)
2007	1161	152(13.09%)	244(21.02%)	291(25.06%)	265(22.83%)	120(10.34%)
2008	1180	225(19.07%)	356(30.17%)	472(40.00%)	411(34.83%)	238(20.17%)
2009	1200	196(16.33%)	310(25.83%)	369(30.75%)	311(25.92%)	144(12.00%)
2010	1260	161(12.78%)	218(17.30%)	221(17.54%)	176(13.97%)	72(5.71%)
2011	1374	172(12.52%)	251(18.27%)	289(21.03%)	218(15.87%)	87(6.33%)
2012	1399	193(13.80%)	270(19.30%)	256(18.30%)	243(17.37%)	122(8.72%)
2013	1586	310(19.55%)	408(25.73%)	419(26.42%)	408(25.73%)	224(14.12%)
2014	1441	199(13.81%)	276(19.15%)	292(20.26%)	275(19.08%)	141(9.78%)
2015	1433	189(13.19%)	258(18.00%)	245(17.10%)	243(16.96%)	120(8.37%)
2016	1443	293(20.30%)	348(24.12%)	349(24.19%)	311(21.55%)	167(11.57%)

Again, we can observe that the normality assumption is more times rejected when we consider inter-day prices, just as before (see Tables 4 and 6). However, in this new framework, the normality assumption is not rejected so many times as before. In fact, the percentage of normality rejections decreases substantially, in the case of Shapiro-Wilk's and Jarque-Bera's tests, from values above the 80% to values below 25% for the $\log(SC_{i+1}/SC_i)$ data (Table 1 vs Table 4) and from values above 90% to values ranging 40%-50% for the $\log(SO_{i+1}/SC_i)$ data (Table 3 vs Table 6). The same reduction in the rejection rate is also observed for the remaining tests and for all the years.

If we compare the normality rejection percentage for intra-day prices ($\log(SC_i/SO_i)$), the decreasing is from more than 75% to less than 20% (Shapiro-Wilk's, Tables 2 and 5). The reduction is also evident for the other tests and, in particular, notice that the Jarque-Bera's test in the previous framework gives the higher rates of rejection but for the trimmed data with the more extreme observations removed, is the test with lower rates os rejection. This outcome was somewhat expected because as already said, the more extreme observations will condition the skewness and the kurtosis of the distribution, meaning that some tests supposed to be more affected by those parameters are indeed

Table 5: Total (and percentage) of samples rejected by each test for normality at 1% level of significance, when the data is trimmed ($\log(SC_i/SO_i)$)

Year	K	PCS	KS	AD	SW	JB
2005	901	79(8.77%)	113(12.54%)	124(13.76%)	132(14.65%)	37(4.11%)
2006	1018	91(8.94%)	134(13.16%)	128(12.57%)	131(12.87%)	49(4.81%)
2007	1161	124(10.68%)	203(17.48%)	235(20.24%)	199(17.14%)	88(7.58%)
2008	1180	178(15.08%)	316(26.78%)	423(35.85%)	373(31.61%)	213(18.05%)
2009	1200	188(15.67%)	284(23.67%)	308(25.67%)	261(21.75%)	102(8.50%)
2010	1260	129(10.24%)	147(11.67%)	132(10.48%)	124(9.84%)	37(2.94%)
2011	1374	150(10.92%)	209(15.21%)	214(15.58%)	180(13.10%)	62(4.51%)
2012	1399	179(12.79%)	225(16.08%)	192(13.72%)	187(13.37%)	62(4.43%)
2013	1586	253(15.95%)	330(20.81%)	299(18.85%)	288(18.16%)	126(7.94%)
2014	1441	165(11.45%)	222(15.41%)	213(14.78%)	196(13.60%)	89(6.18%)
2015	1433	179(12.49%)	243(16.96%)	219(15.28%)	208(14.52%)	69(4.82%)
2016	1443	252(17.46%)	290(20.10%)	279(19.33%)	245(16.98%)	100(6.93%)

strongly affected.

In this framework is also obvious that the departure from normality is stronger and seems more dependent to what happens during the closing periods of the financial markets, see Tables 4 and 6. Even in this set up of trimmed data, we observe that the 2008 year (the year of the financial crisis) and for almost all tests, is again the year with the higher rates of normality rejection, as expected.

4 Final remarks

The results of the data testing allows us to discuss two questions: The normality assumption of the log-returns of the stock prices is still a reasonable assumption? There are differences in the normality assumption of the log-returns in intra-day and inter-day prices? Regarding the first question we can observe that when we remove some of the more extreme observations the normality assumption is reasonable for most of the companies stock prices. That is, some trimming ensures normality, validating the applications of several models, of pricing and multivariate time series, including volatility analysis. On the other hand, it seems reasonable to say that what affects more strongly the non-normality of the logarithm of returns for the stock prices are the "things" that happens when the markets are closed. The focus and results in the paper are also relevant, with due differences, for all kinds of financial times series, e.g., bonds and currencies, validating pricing and forecasting models, including volatility analysis.

Table 6: Total (and percentage) of samples rejected by each test for normality at 1% level of significance, when the data is trimmed ($\log(SO_{i+1}/SC_i)$)

Year	K	PCS	KS	AD	SW	JB
2005	901	500(55.49%)	493(54.72%)	442(49.06%)	468(51.94%)	276(30.63%)
2006	1018	555(54.52%)	549(53.93%)	515(50.59%)	535(52.55%)	341(33.50%)
2007	1161	555(47.80%)	678(58.40%)	694(59.78%)	656(56.50%)	516(44.44%)
2008	1180	592(50.17%)	784(66.44%)	912(77.29%)	831(70.42%)	611(51.78%)
2009	1200	477(39.75%)	621(51.75%)	608(50.67%)	567(47.25%)	336(28.00%)
2010	1260	499(39.60%)	522(41.43%)	425(33.73%)	392(31.11%)	279(22.14%)
2011	1374	594(43.23%)	795(57.86%)	931(67.76%)	824(59.97%)	561(40.83%)
2012	1399	618(44.17%)	686(49.04%)	563(40.24%)	524(37.46%)	362(25.88%)
2013	1586	806(50.82%)	881(55.55%)	780(49.18%)	734(46.28%)	487(30.71%)
2014	1441	627(43.51%)	699(48.51%)	682(47.33%)	678(47.05%)	561(38.93%)
2015	1433	596(41.59%)	679(47.38%)	665(46.41%)	676(47.17%)	521(36.36%)
2016	1443	734(50.87%)	861(59.67%)	864(59.88%)	885(61.33%)	708(49.06%)

References

Anderson, T. and Darling, D. (1952). Asymptotic theory of certain goodness-of-fit criteria based on stochastic processes. *Annals of Mathematical Statistics*, 23(2):193–212.

Anderson, T. and Darling, D. (1954). A test of goodness of fit. *Journal of the American Statistical Association*, 49(268):765–769.

Birnbaum, Z. (1952). Numerical tabulation of the distribution of kolmogorov's statistic for finite sample size. *Journal of the American Statistical Association*, 47(259):425–441.

Black, F. and Scholes, M. (1973). The pricing of options and corporate liabilities. *Journal of Political Economy*, 81(3):637–654.

Jarque, C. and Bera, A. (1987). A test for normality of observations and regression residuals. *International Statistical Review*, 55(2):163–172.

Johansen, S. (1988). Statistical analysis of cointegration vectors. *Journal of Economic Dynamics and Control*, 12(2-3):231–254.

Johansen, S. (1991). Estimation and hypothesis testing of cointegration vectors in gaussian vector autoregressive models. *Econometrica*, 59(6):1551–1580.

Kolmogorov, A. (1933). Sulla determinazione empirica di una legge di distribuzione (on the empirical determination of a distribution law). *Giornale dell'Istituto Italiano degli Attuari*, 4:83–91.

Lewis, P. (1961). Distribution of the anderson-darling statistic. *Annals of Mathematical*

Statistics, 32(4):1118–1124.

Lilliefors, H. (1967). On the kolmogorov-smirnov test for normality with mean and variance unknown. *Journal of the American Statistical Association*, 62(318):399–402.

Massey, F. (1951). The kolmogorov-smirnov test for goodness of fit. *Journal of the American Statistical Association*, 46(253):68–78.

Pearson, K. (1900). On the criterion that a given system of deviations from the probable in the case of a correlated system of variables is such that it can be reasonably supposed to have arisen from random sampling. *Philosophical Magazine Series 5*, 50(302):157–175.

Royston, J. (1982). An extension of shapiro and wilk's w test for normality to large samples. *Journal of the Royal Statistical Society. Series C (Applied Statistics)*, 31(2):115–124.

Royston, J. (1983). A simple method for evaluating the shapiro-francia w' test of non-normality. *Journal of the Royal Statistical Society. Series D (The Statistician)*, 32(3):297–300.

Shapiro, S. and Francia, R. (1972). An approximate analysis of variance test for normality. *Journal of the American Statistical Association*, 67(337):215–216.

Shapiro, S. and Wilk, M. (1965). An analysis of variance test for normality (complete samples). *Biometrika*, 52(3/4):591–611.

Stephens, M. (1974). Edf statistics for goodness of fit and some comparisons. *Journal of the American Statistical Association*, 69(347):730–737.

Stephens, M. (1976). Asymptotic results for goodness-of-fit statistics with unknown parameters. *Annals of Statistics*, 4(2):357–369.