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Generalized class of variance estimators under two-phase sampling for partial information case

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This paper considers a class of generalized estimators for estimating the unknown population variance using two auxiliary variables when the mean of one auxiliary variable may not be available. The expressions for bias and mean square error of the proposed estimators are obtained up to the first order of approximation. Conditions for which the proposed generalized estimator is more efficient than the existing estimators have been derived. Both empirical and simulation studies have also been carried out to analyze the efficiency of the proposed estimators with some existing estimators.

keywords: Population variance, auxiliary variable,exponential estimator,two-phase sampling,mean square error, bias.

1 Introduction

In sample surveys, the auxiliary information is used to obtain estimators for the unknown quantities of a finite population. Graunt (1662) was the first who estimated the population of England using auxiliary information. Under some realistic conditions, the use of auxiliary information provides an efficient estimator for estimating the population variance. We have two situations regarding auxiliary information when auxiliary information is already available at population level then we use Single-phase sampling. Otherwise, two-phase sampling is more appropriate to use. In two-phase sampling, we

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used three situations, when auxiliary information is known, partially known and unknown. Samiuddin and Hanif (2007) have reported these three different situations in two-phase sampling. Roy (2003), Singh et al. (2011) and Khan et al. (2012) have worked on regression and ratio-type estimators for estimating the population mean under partial information case. Further Shabbir and Khan (2013) and Singh et al. (2013) have worked on modified exponential type ratio and product estimators for estimating the population mean.

Let $U = \{U_1, U_2, \cdots, U_N\}$ be a finite population consisting of N units. We are taking two auxiliary variables (x, z) and y is taking as a variable of interest. Now let us assume that the mean information for auxiliary variable x is not known for both first and second phase and the mean information for auxiliary variable z is partially known. Let a simple random sample without replacement (SRSWOR) is drawn in each phase, where the first phase sample of size n_1 is drawn to measure auxiliary variable and the second phase sample n_2 ($n_2 \subset n_1$)is drawn from the first phase sample to measure our study variable. An Unbiased estimator for population variance is defined as:

$$t_0 = s_y^2, \tag{1}$$

The variance of t_0 is,

$$Var(t_0) = \left(\frac{S_y^4}{n_2}\right)[\delta_{400} - 1]. \tag{2}$$

We modified Isaki (1983) ratio-type estimator for population variance when the variance of one auxiliary variable is not available is,

$$t_I = s_{y_2}^2 \frac{s_{x_1}^2}{s_{x_2}^2} \frac{S_z^2}{s_{z_2}^2}, \tag{3}$$

The mean square error (MSE) of t_I is,

$$MSE(t_I) \approx S_y^4 \begin{bmatrix} \frac{1}{n_2}(\delta_{400} - 1) + \left(\frac{1}{n_2} - \frac{1}{n_1}\right)(\delta_{040} - 1) + \frac{1}{n_2}(\delta_{004} - 1) \\ -2\left(\frac{1}{n_2} - \frac{1}{n_1}\right)(\delta_{220} - 1) - 2\frac{1}{n_2}(\delta_{202} - 1) \\ +2\left(\frac{1}{n_2} - \frac{1}{n_1}\right)(\delta_{022} - 1) \end{bmatrix}. \tag{4}$$

Upadhyaya and Singh (1999) proposed the variance estimator using the mean auxiliary variable and we modified its estimator using two auxiliary variables when mean of one auxiliary variable is not available,

$$t_u = s_{y_2}^2 \left(\frac{\bar{x}_1}{\bar{x}_2}\right)\left(\frac{\bar{Z}}{\bar{z}_2}\right) \tag{5}$$

The MSE of t_u is,

$$MSE(t_u) = S_y^4 \begin{bmatrix} \frac{1}{n_2}(\delta_{400} - 1) + \left(\frac{1}{n_2} - \frac{1}{n_1}\right) C_x^2 + \frac{1}{n_2} C_z^2 \\ -2\left(\frac{1}{n_2} - \frac{1}{n_1}\right)\delta_{210} C_x - \frac{2}{n_2}\delta_{201} C_z + 2\rho C_x C_z \end{bmatrix} \quad (6)$$

Singh et al. (2011) proposed the exponential ratio estimator under two-phase sampling for the population variance. The modified form of its estimator using two auxiliary variables when the variance of one auxiliary variable is not available,

$$t_s = s_{y_2}^2 \exp\left(\frac{s_{x_1}^2 - s_{x_2}^2}{s_{x_1}^2 + s_{x_2}^2}\right) \exp\left(\frac{S_z^2 - s_{z_2}^2}{S_z^2 + s_{x_2}^2}\right) \quad (7)$$

The MSE of t_s is,

$$MSE(t_s) \approx S_{y_2}^4 \begin{bmatrix} \frac{1}{n_2}(\delta_{400} - 1) + \left(\frac{1}{n_2} - \frac{1}{n_1}\right)(\delta_{040} - 1) + \frac{1}{n_2}(\delta_{004} - 1) \\ -\left(\frac{1}{n_2} - \frac{1}{n_1}\right)(\delta_{220} - 1) - \frac{1}{n_2}(\delta_{202} - 1) + \left(\frac{1}{n_2} - \frac{1}{n_1}\right)(\delta_{022} - 1) \end{bmatrix} \quad (8)$$

Furthermore, Ahmed et al. (2002),Singh et al. (2009),Yadav and Kadilar (2013),Solanki and Singh (2013),Asghar et al. (2014) and Adichwal et al. (2015), Yasmeen et al. (2018a),Asghar et al. (2018), Al-Jararha and Al-jadeed (2018), Yasmeen et al. (2018b) and Shahzad et al. (2018) have worked on regression and ratio-type estimators for estimating the population variance. This is well evidenced by the lack of literature; there is no significant work on variance estimation, especially, on exponential variance estimation for partial information case. Hence, the motivation of this paper is to develop some exponential ratio-type estimators when the mean of one auxiliary variable is not known. We have many practical scenarios where we need to use partial information case. We have organized our paper in the following sequence. In Section 2, we derived the mathematical equations for bias and mean square error (MSE) of our proposed estimators. In Section 3, real-life data is used for numerical purpose then simulate this data up to 2000 times for showing more efficient results. We accumulate our results in Section 4.

2 Proposed Generalized Exponential Ratio-Type Estimator

Following Singh et al. (2009) and Sanaullah et al. (2016), the exponential ratio-type estimator for the population variance using two auxiliary variables is proposed,

$$t_{1PI} = s_{y_2}^2 \exp\left[1 - \frac{2\bar{x}_2}{\bar{x}_1 + \bar{x}_2}\right] \exp\left[1 - \frac{2\bar{z}_2}{\bar{Z} + \bar{z}_2}\right], \quad (9)$$

This leads to the generalized form as:

$$t_{GPI} = \lambda s_{y_2}^2 \exp\left[\alpha\left(\frac{a\bar{x}_1}{(a-1)\bar{x}_1+\bar{x}_2}-1\right)\right]\exp\left[\beta\left(\frac{b\bar{Z}}{(b-1)\bar{Z}+\bar{z}_2}-1\right)\right],$$

$$or \qquad (10)$$

$$t_{GPI} = \lambda s_{y_2}^2 \exp\left[\alpha\left(\frac{\bar{x}_1-\bar{x}_2}{(a-1)\bar{x}_1+\bar{x}_2}\right)\right]\exp\left[\beta\left(\frac{\bar{Z}-\bar{z}_2}{(b-1)\bar{Z}+\bar{z}_2}\right)\right],$$

Remarks: By putting the different values of λ, α, β, b and a, we may get various exponential ratio-cum-ratio and product-cum-product estimators as new family of t_{GPI}. We may take some examples of exponential ratio-cum-ratio estimators as: For $\lambda = 1$, $\alpha = \beta = 1$ and $a = 2$, $b = 2$, t_{1PI} in (10) is reduced as:

$$t_{1PI} = s_{y_2}^2 \exp\left[\frac{\bar{x}_1-\bar{x}_2}{\bar{x}_1+\bar{x}_2}\right]\exp\left[\frac{\bar{Z}-\bar{z}_2}{\bar{Z}+\bar{z}_2}\right], \qquad (11)$$

For $\lambda = 1$, $\alpha = \beta = 1$ and $a = 1$, $b = 1$, t_{2PI} in (10) is reduced as:

$$t_{2PI} = s_{y_2}^2 \exp\left[\frac{\bar{x}_1-\bar{x}_2}{\bar{x}_2}\right]\exp\left[\frac{\bar{Z}-\bar{z}_2}{\bar{z}_2}\right], \qquad (12)$$

For $\lambda = 1$, $\alpha = \beta = 1$ and $a = 1$, $b = 2$, t_{3PI} in (10) is reduced as:

$$t_{3PI} = s_{y_2}^2 \exp\left[\left\{\frac{\bar{x}_1-\bar{x}_2}{\bar{x}_2}\right\}\right]\exp\left[\left\{\frac{\bar{Z}-\bar{z}_2}{\bar{Z}+\bar{z}_2}\right\}\right], \qquad (13)$$

For $\lambda = 1$, $\alpha = \beta = 1$ and $a = 2$, $b = 1$, t_{4PI} in (10) is reduced as:

$$t_{4PI} = s_{y_2}^2 \exp\left[\left\{\frac{\bar{x}_1-\bar{x}_2}{\bar{x}_1+\bar{x}_2}\right\}\right]\exp\left[\left\{\frac{\bar{Z}-\bar{z}}{\bar{z}_2}\right\}\right], \qquad (14)$$

We may generate many more estimators for different values of a and b . To obtain the bias and mean square error under SRSWOR, let us define the notations,

$$e_0 = \frac{s_{y_2}^2 - S_y^2}{S_y^2}, {e'}_1 = \frac{\bar{x}_1 - \bar{X}}{\bar{X}}, e_1 = \frac{\bar{x}_2 - \bar{X}}{\bar{X}}, {e'}_2 = \frac{\bar{z}_1 - \bar{Z}}{\bar{Z}}, e_2 = \frac{\bar{z}_2 - \bar{Z}}{\bar{Z}},$$

$$s_{y_2}^2 = S_y^2(1+e_0), \bar{x}_1 = \bar{X}\left(1+{e'}_1\right), \bar{x}_2 = \bar{X}(1+e_1), \bar{z}_1 = \bar{Z}\left(1+{e'}_2\right), \bar{z}_2 = \bar{Z}(1+e_2)$$

We may adopt that, $E(e_0) = E({e'}_1) = E({e'}_2) = E(e_2) = 0$, $\theta^* = \frac{1}{n_2} - \frac{1}{n_1}$ We are assuming that the population is large so that the finite population correction is ignored and we may get the following results,

$$E\left(e_0{e'}_1\right) = \frac{\delta_{210}C_x}{n_1}, E(e_0e_1) = \frac{\delta_{210}C_x}{n_2}, E\left(e_0{e'}_2\right) = \frac{\delta_{201}C_z}{n_1},$$

$$E(e_0e_2) = \frac{\delta_{201}C_z}{n_2}, E(e_1e_2) = \frac{\rho_{xz}C_xC_z}{n_2}$$

where,

$$\delta_{pqr} = \frac{\mu_{pqr}}{\mu_{200}^{p/2}\mu_{020}^{q/2}\mu_{002}^{r/2}},\ \mu_{pqr} = \frac{1}{N}\sum_{i}^{N}\left(Y_i-\bar{Y}\right)^p\left(X_i-\bar{X}\right)^q\left(Z_i-\bar{Z}\right)^r.$$

We change (10) into $e's$ for obtaining Bias and MSE,

$$\begin{aligned} t_{GPI} =&\lambda\, S_y^2(1+e_0)\exp\left[-\frac{\alpha({e'}_1-e_1)}{a}\left\{1+\left\{{e'}_1-\frac{({e'}_1-e_1)}{a}\right\}\right\}^{-1}\right] \\ &\exp\left[-\frac{\beta e_2}{b}\left\{(1+\frac{e_2}{b})\right\}^{-1}\right] \end{aligned} \tag{15}$$

We now assume that $\left|\frac{e_1}{a}\right| < 1$ so that we may expand $\left(1+\frac{e_1}{a}\right)^{-1}$ and $\left(1+\frac{e_2}{b}\right)^{-1}$ as a series in powers of e_1 and e_2. We expand the exponentials and neglect higher order terms in e_0 and e_1,

$$\begin{aligned} t_{GPI} = \lambda\, S_y^2(1+e_0)\exp&\left[-\frac{\alpha({e'}_1-e_1)}{a}\begin{Bmatrix}1-\left\{{e'}_1-\dfrac{({e'}_1-e_1)}{a}\right\} \\ +\left\{{e'}_1-\dfrac{({e'}_1-e_1)}{a}\right\}^2----\end{Bmatrix}\right] \\ \exp&\left[\left\{1-\frac{e_2}{b}+\frac{e_2^2}{b^2}---\right\}\right] \end{aligned} \tag{16}$$

The bias and the MSE of the proposed estimator is,

$$Bias(t_{GPI}) \approx S_y^2\left[(\lambda-1)+\lambda\begin{Bmatrix}(\dfrac{\alpha}{a^2}+\dfrac{\alpha^2}{2a^2})\theta^*\dfrac{C_x^2}{n}+(\dfrac{\beta}{b^2}+\dfrac{\beta^2}{2b^2})\dfrac{C_z^2}{n_2}-\dfrac{\alpha}{a}\theta^*\delta_{210}C_x \\ -\dfrac{\beta}{n_2 b}\delta_{201}C_z+\dfrac{\alpha\beta}{ab}\theta^*\rho C_x C_z\end{Bmatrix}\right] \tag{17}$$

and

$$MSE(t_{GPI}) \approx S_y^4\left[\begin{matrix}\left\{1+\lambda^2\begin{Bmatrix}1+\dfrac{(\delta_{400}-1)}{n_2}+2(\dfrac{\alpha}{a^2}+\dfrac{\alpha^2}{a^2})\theta^*C_x^2+2(\dfrac{\beta}{b^2}+\dfrac{\beta^2}{b^2})\dfrac{1}{n_2}C_z^2 \\ -\dfrac{4\alpha}{a}\theta^*\delta_{210}C_x-\dfrac{4\beta}{bn_2}\theta^*\delta_{201}C_z+4\dfrac{\alpha\beta}{ab}\theta^*\rho C_x C_z\end{Bmatrix}\right\} \\ -2\lambda\begin{Bmatrix}1+(\dfrac{\alpha}{a^2}+\dfrac{\alpha^2}{2a^2})\theta^*C_x^2+(\dfrac{\beta}{b^2}+\dfrac{\beta^2}{2b^2})\dfrac{1}{n_2}C_z^2 \\ -\dfrac{\alpha}{a}\theta^*\delta_{210}C_x-\dfrac{\beta}{bn_2}\delta_{201}C_z+\dfrac{\alpha\beta}{ab}\theta^*\rho C_x C_z\end{Bmatrix}\end{matrix}\right] \tag{18}$$

The minimum MSE of t_{GPI} at optimum value of λ is,

$$\lambda_{GPI} = \frac{\begin{bmatrix} 1+(\frac{\alpha}{a^2}+\frac{\alpha^2}{2a^2})\theta^* C_x^2 + (\frac{\beta}{b^2}+\frac{\beta^2}{2b^2})\frac{1}{n_2}C_z^2 \\ -\frac{\alpha}{a}\theta^*\delta_{210}C_x - \frac{\beta}{bn_2}\delta_{201}C_z + \frac{\alpha\beta}{ab}\theta^*\rho C_x C_z \end{bmatrix}}{\begin{bmatrix} 1+\frac{(\delta_{400}-1)}{n_2}+2(\frac{\alpha}{a^2}+\frac{\alpha^2}{a^2})\theta^* C_x^2 + 2(\frac{\beta}{b^2}+\frac{\beta^2}{b^2})\frac{1}{n_2}C_z^2 \\ -\frac{4\alpha}{a}\theta^*\delta_{210}C_x - \frac{4\beta}{bn_2}\delta_{201}C_z + 4\frac{\alpha\beta}{ab}\theta^*\rho C_x C_z \end{bmatrix}}$$

$$or \quad \lambda_{GPI} = {}^{A}\!/_{B}$$

where,

$$A = \begin{bmatrix} 1+(\frac{\alpha}{a^2}+\frac{\alpha^2}{2a^2})\theta^* C_x^2 + (\frac{\beta}{b^2}+\frac{\beta^2}{2b^2})\frac{1}{n_2}C_z^2 \\ -\frac{\alpha}{a}\theta^*\delta_{210}C_x - \frac{\beta}{bn_2}\delta_{201}C_z + \frac{\alpha\beta}{ab}\theta^*\rho C_x C_z \end{bmatrix}$$

$$B = \begin{bmatrix} 1+\frac{(\delta_{400}-1)}{n_2}+2(\frac{\alpha}{a^2}+\frac{\alpha^2}{a^2})\theta^* C_x^2 + 2(\frac{\beta}{b^2}+\frac{\beta^2}{b^2})\frac{1}{n_2}C_z^2 \\ -\frac{4\alpha}{a}\theta^*\delta_{210}C_x - \frac{4\beta}{bn_2}\delta_{201}C_z + 4\frac{\alpha\beta}{ab}\theta^*\rho C_x C_z \end{bmatrix}$$

and

$$MSE(t_{GPI)\min} \approx S_y^4 \left[1 - \frac{\begin{bmatrix} 1+(\frac{\alpha}{a^2}+\frac{\alpha^2}{2a^2})\theta^* C_x^2 + (\frac{\beta}{b^2}+\frac{\beta^2}{2b^2})\frac{1}{n_2}C_z^2 \\ -\frac{\alpha}{a}\theta^*\delta_{210}C_x - \frac{\beta}{bn_2}\delta_{201}C_z + \frac{\alpha\beta}{ab}\theta^*\rho C_x C_z \end{bmatrix}^2}{\begin{bmatrix} 1+\frac{(\delta_{400}-1)}{n_2}+2(\frac{\alpha}{a^2}+\frac{\alpha^2}{a^2})\theta^* C_x^2 + 2(\frac{\beta}{b^2}+\frac{\beta^2}{b^2})\frac{1}{n_2}C_z^2 \\ -\frac{4\alpha}{a}\theta^*\delta_{210}C_x - \frac{4\beta}{bn_2}\delta_{201}C_z + 4\frac{\alpha\beta}{ab}\theta^*\rho C_x C_z \end{bmatrix}} \right] \tag{19}$$

On substituting the optimal values of λ , we may get the asymptotically optimal estimator,

$$t_{asym} = {}^{A}\!/_{B}\, s_{y_2}^2 \exp\left[\alpha\left(\frac{a\bar{x}_1}{(a-1)\bar{x}_1+\bar{x}_2}-1\right)\right]\exp\left[\beta\left(\frac{b\bar{Z}}{(b-1)\bar{Z}+\bar{z}_2}-1\right)\right], \tag{20}$$

In some situations, we replace λ, by its consistent estimates because it is not possible to presume the values of λ practically, the estimator in (10) may be obtained as:

$$\hat{t}_{asym} = {}^{\hat{A}}\!/_{\hat{B}}\, s_{y_2}^2 \exp\left[\alpha\left(\frac{a\bar{x}_1}{(a-1)\bar{x}_1+\bar{x}_2}-1\right)\right]\exp\left[\beta\left(\frac{b\bar{Z}}{(b-1)\bar{Z}+\bar{z}_2}-1\right)\right], \tag{21}$$

Similarly the $MSE\left(t_{GPI}\right)$ in (19) may be given as:

$$MSE(t_{NG})_{\min} \approx S_y^4\left[1-\frac{\hat{A}^2}{\hat{B}}\right]$$

Thus, the estimator $\hat{t}_{asym}$ given in (21), is to be used in practice. One can obtain many values of λ_{GPI} and minimum MSE's by choosing the different values of a and b, For $\lambda=\lambda_{1PI}$, $\alpha=\beta=1$ and $a=2$, $b=2$, t_{1PI} in (11) is reduced as:

$$MSE(t_{1PI})_{\min} \approx S_y^4\left[1-\frac{\left\{\begin{array}{l}1+\frac{3}{8}\theta^* C_x^2+\frac{3}{8n_2}C_z^2-\frac{1}{2}\theta^*\delta_{210}C_x\\ -\frac{1}{2n_2}\delta_{201}C_z+\frac{1}{4}\theta^*\rho C_x C_z\end{array}\right\}^2}{\left\{\begin{array}{l}1+\frac{(\delta_{400}-1)}{n_2}+\theta^* C_x^2+\frac{1}{n_2}C_z^2\\ -2\theta^*\delta_{210}C_x-\frac{2}{n_2}\delta_{201}C_z+\theta^*\rho C_x C_z\end{array}\right\}}\right] \tag{22}$$

For $\lambda=\lambda_{2PI}$, $\alpha=\beta=1$ and $a=1$, $b=1$, t_{2PI} in (12) is reduced as:

$$MSE(t_{2PI})_{\min} \approx S_y^4\left[1-\frac{\left\{\begin{array}{l}1+\frac{3}{2}\theta^* C_x^2+\frac{3}{2}\frac{1}{n_2}C_z^2-\theta^*\delta_{210}C_x\\ -\frac{1}{n_2}\delta_{201}C_z+\theta^*\rho C_x C_z\end{array}\right\}^2}{\left\{\begin{array}{l}1+\frac{(\delta_{400}-1)}{n_2}+4\theta^* C_x^2+4\frac{1}{n_2}C_z^2\\ -4\theta^*\delta_{210}C_x-\frac{4}{n_2}\delta_{201}C_z+4\theta^*\rho C_x C_z\end{array}\right\}}\right] \tag{23}$$

For $\lambda=\lambda_{3PI}$, $\alpha=\beta=1$ and $a=1$, $b=2$, t_{3PI} in (13) is reduced as:

$$MSE(t_{3PI})_{\min} \approx S_y^4\left[1-\frac{\left\{\begin{array}{l}1+\frac{3}{2}\theta^* C_x^2+\frac{3}{8n_2}C_z^2-\theta^*\delta_{210}C_x\\ -\frac{1}{2n_2}\delta_{201}C_z+\frac{1}{2}\theta^*\rho C_x C_z\end{array}\right\}^2}{\left\{\begin{array}{l}1+\frac{(\delta_{400}-1)}{n_2}+4\theta^* C_x^2+\frac{1}{n_2}C_z^2\\ -4\theta^*\delta_{210}C_x-\frac{2}{n_2}\delta_{201}C_z+2\theta^*\rho C_x C_z\end{array}\right\}}\right] \tag{24}$$

For $\lambda = \lambda_{4PI}$, $\alpha = \beta = 1$ and $a = 2$, $b = 1$, t_{4PI} in (14) is reduced as:

$$MSE(t_{4PI})_{\min} \approx S_y^4 \left[1 - \frac{\left\{ \begin{array}{l} 1 + \frac{3}{8}\theta^* C_x^2 + \frac{3}{2n_2} C_z^2 - \frac{1}{2}\theta^* \delta_{210} C_x \\ -\frac{1}{n_2}\delta_{201} C_z + \frac{1}{2}\theta^* \rho C_x C_z \end{array} \right\}^2}{\left\{ \begin{array}{l} 1 + \frac{(\delta_{400} - 1)}{n_2} + \theta^* C_x^2 + 4\frac{1}{n_2} C_z^2 \\ -2\theta^* \delta_{210} C_x - \frac{4}{n_2}\delta_{201} C_z + 2\theta^* \rho C_x C_z \end{array} \right\}} \right] \quad (25)$$

3 Numerical Illustration

In order to examine the performance of proposed estimators under two-phase sampling real-life data from Cochran (1977) has been used where we consider measure Y the Food cost,X for Size and Z for Income. The following information is given by:

$$N = 33,\ n_1 = 9,\ n_2 = 4,\ C_y = 0.362887$$
$$\delta_{201} = 0.5506, \delta_{040} = 2.380, \delta_{220} = 1.430$$
$$\delta_{202} = 2.255, \delta_{004}, = 2.143, \delta_{022} = 1.492$$
$$C_x = 0.143577, \rho_{yx} = 0.42378, \rho_{yz} = 0.2521$$
$$\rho_{xz} = -0.065989, \delta_{400} = 5.72, \delta_{210} = 0.6305$$

We have computed the percent relative efficiencies (PREs) of t_{GPI} with respect to t_0 using the following formula.

$$PREs\,(t_i,\, t_0)\ =\ \frac{Var(t_0)}{MSE(t_i)} \times 100$$

Following Abu-Dayyeh and Ahmed (2005) we have also computed the simulation study for showing more efficient results (in terms of less MSE for our proposed estimator). The procedure for simulation study to find the MSE of the estimators is given below:
i- Total number of units in the population N =33.
ii- Select a simple random sample (SRSWOR) of size $n1 = 9$ from the real data-set of size $N = 33$.
iii- Select a simple random sample (SRSWOR) of size $n_2 = 4$ from $n_1 = 9$ selected in step one.
iv- Use the samples in step (ii) and (iii) to find the estimator t_{GPI}.
v- Repeat steps (ii)-(iv) for k=2000 times. Then, we obtain $k = 2000$ values for each estimator.
vi- The MSE of t_{GPI} is obtained as:
$MSE(t_i) = \frac{1}{k} \sum\limits_{j=1}^{k} (t_{ij} - S_y^2)^2,\ where, i = 0, I, u, s, 1PI, 2PI, 3PI, 4PI$

Table 1: Empirical results

Estimator	PRE's
t_0	100
t_I	112.688
t_u	115.4435
t_s	87.26018
t_1PI	218.0471
t_2PI	222.6224
t_3PI	225.0929
t_4PI	216.4012

Table 2: Simulated results

Estimator	PRE's
t_0	100
t_I	118.3011
t_u	101.6951
t_s	107.8068
t_1PI	173.2784
t_2PI	460.9489
t_3PI	411.9863
t_4PI	185.2351

4 Conclusion

It is observed that the proposed class of estimators $((t_{1PI}),(t_{2PI}),(t_{3PI}),(t_{4PI}))$ are performing better than usual unbiased estimator (t_0), modified Isaki (1983) (t_I), Upadhyaya and Singh (1999) (t_u) and Singh et al. (2011)(t_s) from the empirical study as shown in Table 1. We also perform the simulation study to check the performance of our class of estimators and find that our proposed estimators are also more efficient than the existing estimators as shown in Table 2. Furthermore, it is also observed that the class of exponential type ratio-cum-ratio estimators t_2PI and t_3PI are more efficient than the existing estimators in both studies.

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