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A new hybrid approach EMD-EXP for short-term forecasting of daily stock market time series data

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Forecasting time series recently has attracted considerable attention in the field of analyzing financial time series data specifically stock market index. This considerable attention confined itself in the need of transparent change in the governmental policies whether attracting foreign investment or/and economical advancements. In this study, a hybrid methodology between Empirical Mode Decomposition with exponential smoothing method (EMD-EXP) is used to improve forecasting performances in financial time series. The strength of this EMD-EXP lies in its ability to predict non-stationary and non-linear time series without need to use any transformation method. Moreover, EMD-EXP also has relatively high accuracy and offer a new forecasting method in time series. The daily stock market time series data of 12 countries are used to show the forecasting performance of the proposed EMD-EXP. Based on the three forecast accuracy measures, the results indicate that EMD-EXP forecasting performance is superior to seven traditional forecasting methods.

keywords: forecast time series, empirical mode decomposition (EMD), exponential smoothing forecasting (EXP), intrinsic mode function (IMF), seasonal-trend decomposition (STL).

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1 Introduction

In financial time series analysis, one of the primary issues is modeling and forecasting financial time data specifically stock market index. Usually, the transformation of a financial time series, rather than its original scale, is taken for describing its dynamics. Proper transformation is necessary to convert original non-stationary processes to stationary processes and subsequently to utilize mathematical and statistical properties for stationary processes.

In general, time series forecasting method has effectively solved forecasting problems in financial time series. Unfortunately, there still have three major problems with forecasting method of financial time series as presented in Oh et al. (2009) and Cheng and Wei (2014). The first problem, there is data sets not following the statistical assumptions. Thus, cannot be used it on some of forecasting methods. The second problem is the changes in market conditions, for example Supply and Demand environments have been given many noises involuted to stock market time series data. This would weaken the forecasting execution in most time series forecasting methods. The third problem is the assumptions of stationarity and linearity are not always true for some financial time series data. In order to solve the aforementioned problems an advancing the performance of time series forecasting methods are needed. Therefore, this paper proposes forecasting method based on exponential smoothing (EXP) forecasting with empirical mode decomposition (EMD) to overcome the three problems.

Recently the empirical mode decomposition has been employed to forecast time series by several of previous researchers in various fields. This method was used to decompose the non-stationary, non-linear, high frequency or/and fuzzy time series data into IMFs and residual components where later use forecasting model to forecast. Then all these forecasted values were aggregated to produce the final forecasted value of the original time series. Such as in Abadan and Shabri (2014) used a hybrid EMD-ARIMA to forecasting the monthly prices of rice data. The authors applied EMD to the original time series data to decompose into IMFs and residue and applied the ARIMA model to predict each IMF and the residue. After that, the outcomes were collected together to obtain the forecast. Then, Li and Wang (2008) also used the same methodology, but with wind speed data.

A hybrid EMD-LSSVR (least squares support vector regression) forecasting model has been applied on foreign exchange rate in Lin et al. (2012). This model outperforms the EMD-ARIMA, LSSVR and ARIMA models without time series decomposition. This model used EMD to get IMFs and residual components and then applied least squares support vector regression (LSSVR) to forecast each IMFs and residual. After constructing it according to IMFs and residual value individually, the forecasting results have been aggregated to produce the final forecasting value for original time series. While in Tatinati and Veluvolu (2013) used a hybrid of EMD, LS-SVM (Least Squares-Support Vector Machines), and AR model with Kalman filter to predict wind speed data. The results showed that this method provides better forecasting compared to the existing methods. This model used EMD to get IMFs after that employed Multistep prediction, Least squares-support vector machines and autoregressive model with a Kalman filter.

Based on the partial autocorrelation factor of the individual IMFs, after that computed the aggregate of IMFs prediction.

EMD-FNN-ALNN ensemble learning approach has been applied on two main crude oil price series in Yu et al. (2008). In order to verify the effectiveness of this approach, EMD has been used to decompose the original time series into the IMFs and residual components. Then a three-layer feed-forward neural network (FNN) has been used to predict each component and the results has been combined with an adaptive linear neural network (ALNN) to formulate an ensemble output for the original time series. However, in Guo et al. (2012) the EMD of the wavelet transform has been introduced and applied on the time series of oil futures at 5 minute intervals. The data has been decomposed by EMD and then the evolutionary law and development trend of each component of the IMF has been explored in different time scales. After that, the forecast model has been reconstructed by using the IMFs components.

A novel EMD-BPN model has been applied to time series of tourism demand in Chen et al. (2012). The EMD has been used to decompose the original data into IMFs and a residue. These IMFs and residue have been modeled and forecasted using back-propagation neural network (BPN) and the final forecasting value has been obtained by the sum of these prediction results. According to their results, the EMD-BPN model outperforms the single BPN model without EMD and the traditional (ARIMA) models. While in Guo et al. (2012) applied EMD-FNN forecasting model on the mean monthly and daily wind speed in Zhangye of China. The original series has been decomposed into IMFs and residual by using the EMD. Then these components are extracted except the high frequency have been forecasted by feed-forward neural network (FNN). After that the predicted results of the modeled IMFs and residual series have been summed to formulate an ensemble forecast for the original series. The result indicates that EMD-FNN model has the best accuracy compared with the basic FNN and unmodified EMD-FNN.

With regard to all those literature reviews, this study attempts to apply a hybrid of EMD-EXP to forecast the daily stock market data of 12 countries. In order to assess the performance of forecasting, and the proposed method is compared with the forecast of seven traditional forecasting methods. Experimental results show that the proposed method is superior to existing method in terms of three accuracy forecasting measure. Section 2 introduces methods is used in methodology in this paper which are EMD, IMF, exponential smoothing and the STL decomposition. Section 3 presented the proposed methodology with flowchart explain the steps. Section 4 analyzes The daily stock market time series data of 12 countries with a discussion the result showing the capability of EMD-EXP. Finally, in Section 5 some concluding remarks are addressed.

2 Methodology

In this section, the various steps for the implementation of the EMD-EXP forecasting method are described in detail. Which are Empirical Mode Decomposition, exponential smoothing and the STL decomposition.

2.1 Empirical mode decomposition[EMD]

EMD has been described by Huang et al. (1998) and Rilling et al. (2003) and this method has been modified by Deering and Kaiser (2005), Lu et al. (2013) and Jaber et al. (2013). EMD has been included application in different fields in sciences such as financial by Jaber et al. (2014), Awajan et al. (2017b), Awajan and Ismail (2017) and Awajan et al. (2017a), biological by Huang et al. (2002), climate by Coughlin and Tung (2004) and dynamic by Zhang et al. (2003). The main idea of EMD is decompose of nonlinear and non-stationary time series data $x(t)$ -with keeping the time domain of the signal- into n of simple time series that known as Intrinsic Mode Functions (IMF). Later, the original signal can be constructed back as the following:

$$x(t) = \sum_{i=1}^{n} IMF_i(t) + r(t) \tag{1}$$

where $r(t)$ represents the residue of the original time series data decomposition and IMF_i represent the i^{th} IMF time series.

In order to estimate these IMFs, the following steps should be initiated and the process is called the sifting process of time series $x(t)$ are shown below:

1. Start the first step by taking the original time series $x(t)$ for sifting process and assuming the iteration index value is $i = 1$
2. Then, evaluate all of local extrema value of the time series $x(t)$.
3. After that, form the local maxima (local upper) envelope function $u(t)$ by connecting all local maxima value using a cubic spline line. In a similar way, form the local minimum (local lower) envelope function $l(t)$, and then form the mean function $m(t)$ by using this following;
$$m(t) = \frac{u(t) + l(t)}{2} \tag{2}$$
4. Next defined as a new function $h(t)$ using the mean envelope $m(t)$ and the signal $x(t)$ on this formula
$$h(t) = x(t) - m(t) \tag{3}$$
Check the function $h(t)$ is an IMF, according to IMF conditions (shown in the second part of this section). If the function $h(t)$ has satisfied IMF conditions continue to step 5. If not, go back to step 2 and renew the value of $x(t)$ such that became $h(t)$, repeat again steps 2 until 4.
5. In step 5, firstly saves the result of the IMF obtain from the last step. Secondly, renew the iteration index value such that became $i = i + 1$. Thirdly attain the residue function $r(t)$ using the IMF and the signal $x(t)$ on the formula.
$$IMF_i(t) = h(t) \Rightarrow r_{i+1}(t) = x(t) - IMF_i(t) \tag{4}$$

6. Finally made a decision whether the residue function $r(t)$ that acquire from step 5 is a monotonic or constant function. Then, save the residue and all the IMFs obtained. If the residue is not monotonic or constant function, return to step 2.

The steps 1 to 6 which was discussed above allow the sifting process (EMD algorithm) to separate time-altering signal features. Figure 1 summarizes all steps of sifting process.

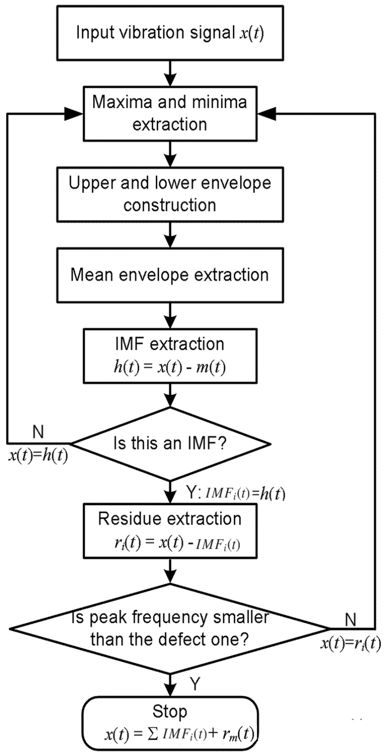

Figure 1: Flowchart of empirical mode decomposition estimation process

2.2 Intrinsic Mode Function [IMF]

Based on the EMD algorithm presented in the previous section, the IMF produces by the sifting process need to satisfy two conditions which are

1. $$|N[EX] - N[CZ]| < 1 \tag{5}$$

   where N[EX] represents the number of local extreme points (all local maxima and all local minima), also N[CZ] represents the number of cross-zero points.

2. $$|m(t)| = |\frac{u(t) + l(t)}{2}| < \varepsilon \tag{6}$$

where $u(t)$ represents the envelope function generated by using cubic spline line on all local maxima, $l(t)$ represents the envelope function generated by using a cube spline line on all local minima, $m(t)$ represents the mean function that it obtained by evaluating the mean of $u(t)$ and $l(t)$, and ε is a very small positive number that close to zero, sometime equal zero.

2.3 A Seasonal-Trend Decomposition Using Loess [STL]

Persons (1919) seems to be the first that developed decomposition techniques for identifying and isolating the salient features of a time series. He proposed this technique to decompose time series into four components: the trend, the cycle, seasonality and a purely accidental hazard. Presently, there are several methods of decompose time series. The most prominent methods are X-11 ARIMA/88 in Dagum (1988), Seasonal Adjustment at Bell Laboratories [SABL] Cleveland et al. (1981) and Seasonal-Trend decomposition based on Loess smoothing [STL] in Cleveland et al. (1990).

STL is a robust nonparametric time series decomposition method known as the Seasonal-Trend decomposition filtering procedure based on Loess was introduced by Cleveland et al. (1990). STL can divide the time series data into three additive main components: Seasonal $S(t)$, Trend or long-term $T(t)$, and Random $R(t)$ components, i.e. $Y(t) = S(t) + T(t) + R(t)$. The trend is the general tendency of a time series to increase, decrease or stagnate over a long period of time. While seasonal in a time series are fluctuations within a year during the season, such as the effect of weather on sales of clothes. Seasonality is always of a fixed and known period. The random in a time series are caused by unpredictable influences, which are not regular and also do not repeat in a particular pattern. Loess method (a smoothing method based on local regressions) are used to decompose the time series into trend and remainder components for non-seasonal time series, as presented by Cleveland et al. (1992). There are a number of papers in the literature which applied STL decomposition in forecasting time series through used it as a step in his methodology. Such as Theodosiou (2011) and Yang et al. (2015).

2.4 Exponential Smoothing Methods

More than fifty five years ago, the basic formula of the exponential-smoothing family have been presented by Holt (1957) and Winters (1960), where it was known as Holt-Winters. This method is simple, does not need high data-storage requirements, and is easily automated. Moreover, this method also is the favorite until this present day to be implemented in several of forecasting financial time series studies. In this method the recent observations have effect more robustly than old observations in forecasting value.

The exponential smoothing methods have been classified in Pegels (1969) according to the trend and seasonal component based on whether they are additive or multiplicative. After that, this classification have been extended by Gardner (1985) to include damped trends. Recently, the exponential smoothing methods have been extended by Gardner (2006) and he suggested fifteen exponential smoothing methods. Where each method has two components, seasonality and trend as given in the following Table (1).

Table 1: Exponential smoothing methods

Trend Component	Seasonal Component		
	N(None)	A(Additive)	M(Multiplicative)
N(None)	N,N	N,A	N,M
A(Additive)	A,N	A,A	A,M
Ad(Additive damped)	A_d,N	A_d,A	A_d,M
M(Multiplicative)	M,N	M,A	M,M
M_d(Multiplicative damped)	M_d,N	M_d,A	M_d,M

Exponential smoothing models may be different according on the presence of the two components, trend and seasonal as presented in Table (1). However, in the studies of Hyndman and Athanasopoulos (2014), they have extended this classification and showed that there are two underlying innovation state space model. Model with additive errors (A) and model with multiplicative errors (M) for each of the fifteen models in Table (1). Thus, the present classification had produced 30 exponential smoothing methods.

To describe those models with (A,M) errors, we need three letters to the front of the method notation. The triplet (E,T,S) refers to the three components: Error, Trend and Seasonality. For example the additive error model ETS(A,A,A) represent the the additive Holt-Winters method, defined by the following:

$$l_t = \alpha(y_t - s_{t-m}) + (1-\alpha)(l_{t-1} + b_{t-1})$$

$$b_t = \beta(l_t - l_{t-1}) + (1-\beta)b_{t-1}$$

$$s_t = \gamma(y_t - l_{t-1} - b_{t-1}) + (1-\gamma)s_{t-m}$$

$$\hat{y}_{t+h/t} = l_t + hb_t + s_{t-m+h_m^+}$$

such that, l_t represent the level of series at time t, b_t represent the slope (growth) at time t, s_t represent the seasonal component of the series at time t, and m represent the number of seasons in a year. The constants α, β and γ are smoothing parameters in the [0,1]-interval, h is the forecast horizon, and $h_m^+ = [(h-1) * mod(m)] + 1$. This method uses the maximum likelihood function to estimate the starting parameters and then it may estimate iteratively all the parameters to forecasting future values of time series.

The ETS framework provides an automatic way of selecting the best method, using the bias-corrected AIC. The model identification criterion from ETS family is the Akaike Information Criterion (AIC). It consists on defining the function

$$AIC(p) = -2log(L(p)) + 2p$$

where $L(p)$ maximum likelihood function and select p such that minimizes $AIC(p)$. According to this criterion, p is the optimal value that minimizes the $AIC(p)$. This criterion helps in selecting the most appropriate model: model A is said to fit better than model B if the AIC value for A is smaller than for model B.

2.5 Statistical techniques for consideration method

In this study the hyper EMD-EXP method is compered with seven method. EXP, EXP-STL without EMD are presented in section 2, ARIMA models are presented in Peiris and Perera (1988), Structural Time Series is presented in Harvey and Peters (1990), Theta method is presented in Assimakopoulos and Nikolopoulos (2000), Holt-Winters method is presented in Winters (1960) and Holt (1957) and Random Walk method are used in order to validate the forecasting performance of EMD-EXP.
These statistical methods were selected based on their performance in forecasting competitions and other empirical applications, as well as on their ability to capture salient features of the data. ARIMA (autoregressive integrated moving average) or Box-Jenkins models are generally denoted ARIMA(p, d, q). Where parameters p, d, and q are non-negative integers, p is the order of the Autoregressive model, d is the degree of differencing, and q is the order of the Moving-average model. HW (Holt-Winters method) has been examined in Rossi and Brunelli (2015) and was reported to perform well for of two different electricity data of centers power consumption. Theta method is presented in Assimakopoulos and Nikolopoulos (2000). But by Hyndman and Billah (2003) presented that the forecasts obtained of theta method are equivalent to simple exponential smoothing with drift. And showed this method is the best performing method in the M3-Competition Makridakis and Hibon (2000). Structural Time Series is applied by Turner and Witt (2001) on Inbound tourism to New Zealand from selected countries. This method has outperformed the naive process. A random walk (RW) is a process where the current value of a variable is composed of the past value with adding an error term defined as a white noise. It was first studied several hundred years ago as models for games of chance. Recently, In Muck and Skrzypczynski (2012) have been shown that the random walk model turns out to be a hard to beat benchmark in forecasting the CEE exchange rates. And in Rowland et al. (2003) applied a variable drift term with the random walk process. This was estimated using a Kalman filter. This simple statistical process was shown to perform better than all the three models that he was selected in out of-sample forecasts.

3 Propose Methodology and Data

This section contains three parts. The first part is about the data that is used to implement the proposed methodology. While the second part presents the proposed methodology EMD-EXP with detailed description of each step by using Malaysia Stock Market Index data (KLSM). The last part, presents the algorithm of propose methodology.

3.1 Data

In this study, nonlinear and non-stationary time series data from the daily stock market of 12 countries are used. The countries are Belgium, Denmark, Estonia, France, Germany, India, Japanese, Malaysia, New Zealand, Saudi, United Kingdom, USA S&P500. Table 2 presents the Basic statistics and the number of observations for each country.

Where S.Deviation = Standard Deviation and N = the number of observations. The data are extracted from the Yahoo finance website (www.finance.yahoo.com). Figure 2 shows the time series plots of the stock market time series data of these countries. The stock market data are chosen because these data non-linear and non-stationary. Moreover, these data do not satisfy the balanced condition and, therefore, cannot be forecasted by traditional methods. Thus, it is meaningful to explore other effective forecasting methods. The daily closing prices are used as a general measure of the stock market over the past six years. The whole data set -for each country- covers the period from 9 February 2010 to 7 January 2016 with N observations that is presented in Table 2 for each country. The data set is divided into two parts. The first part (n observations) is used to determine the specifications of the models and parameters. The second part, on the other hand, (h observations) is reserved for out-of-sample evaluation and comparison of performances among various forecasting models. Malaysian stock market data (KLSM) are taken as example where N= 1459. On the other hand of h observation where h=2, 3, 4 and 5. The first part of n observations where n = 1457, 1456, 1455 and 1454 respectively, are used.

Table 2: Basic statistics

Country	Mean	Median	S.Deviation	Skewness	Kurtosis	N
Belgium	2785.21	2658.78	484.60	0.55	-0.64	1517
Denmark	586.71	519.34	197.16	0.84	-0.52	1466
Estonia	726.91	759.33	112.62	-0.27	-1.29	1471
France	3968.26	3939.82	557.54	0.21	-0.60	1516
Germany	8102.02	7637.87	1791.81	0.45	-0.91	1510
India	20964.54	19501.08	4045.57	0.73	-0.94	1465
Japanese	12835.70	11182.58	3831.53	0.56	-1.04	1450
Malaysia	1638.20	1643.89	164.52	-0.40	-0.68	1459
New Zealand	4317.80	4340.73	1024.04	0.28	-1.47	1419
Saudi Arabia	7718.91	7189.13	1343.88	0.64	-0.77	1287
UK	6114.40	6059.30	538.36	-0.15	-1.09	1528
US S.P500	1579.25	1493.69	344.31	0.20	-1.44	1490

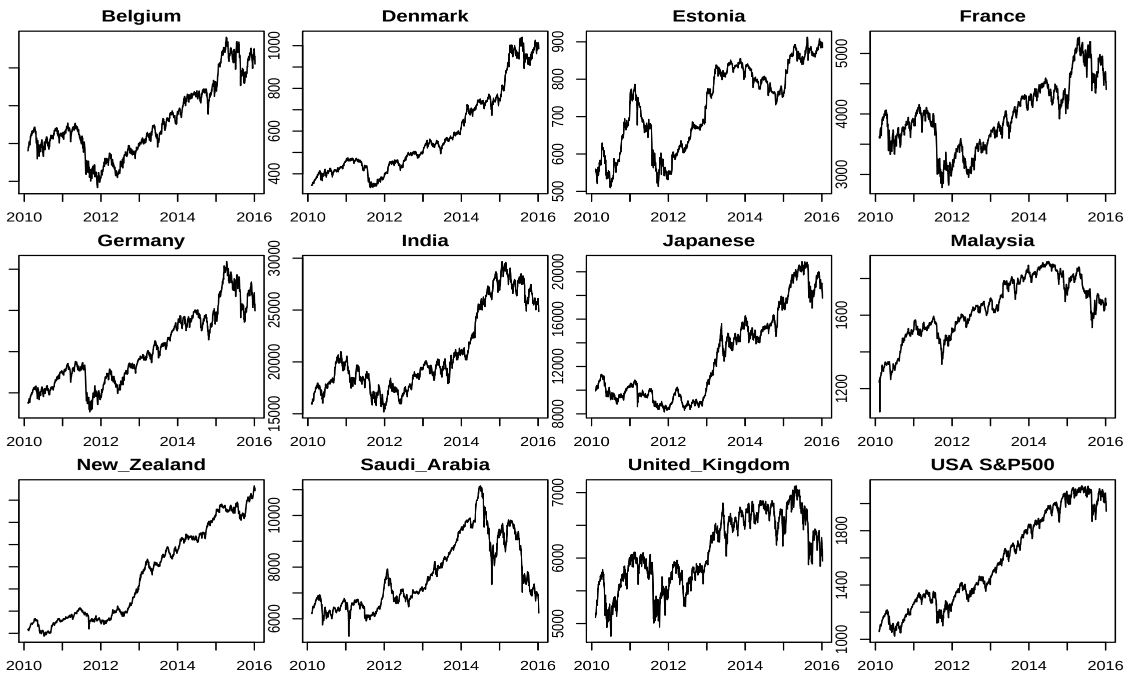

Figure 2: Time series plots

3.2 Propose Methodology

The proposed methodology consists of four stages. The Malaysian stock market data (Kuala-Lumpur stock market (KLSM)) are used as an example to illustrate the steps. Firstly, the use of Empirical Mode Decomposition (EMD) on the KLSM. In this stage, seven Intrinsic Mode Functions (IMFs) and residue are obtained. The results are displayed in Figure 3. Secondly, the STL decomposition is applied to decompose each IMF and residue into (seasonal, trend and random) component. Figure 4 depicts an example of the results from the application of the STL decomposition procedure to the IMF(3) which was obtained from the results of EMD on KLSM. Thirdly, according to the results of second stage for each IMF and residue. The Exponential Smoothing Model (EXP) is selected from ETS family. This model is used to forecast h days ahead. For example, the forecasting result for IMF(3) is shown in Figure 5. Finally, in the last stage all the forecasted results for IMFs and residue are added up. This methodology is presented as a flowchart in Figure 6.

3.3 Algorithm of Propose Methodology

To summarize, a scheme of the EMD-EXP procedure is given in Algorithm 1. At first, EMD is used to decompose the original time series to IMFs and residue. And the each IMF an residue are decomposed into trend, seasonal part and remainder, using STL or loess. An ETS model is fitted and used for forecasting each components of IMFs and residue, the forecasting results are added together.

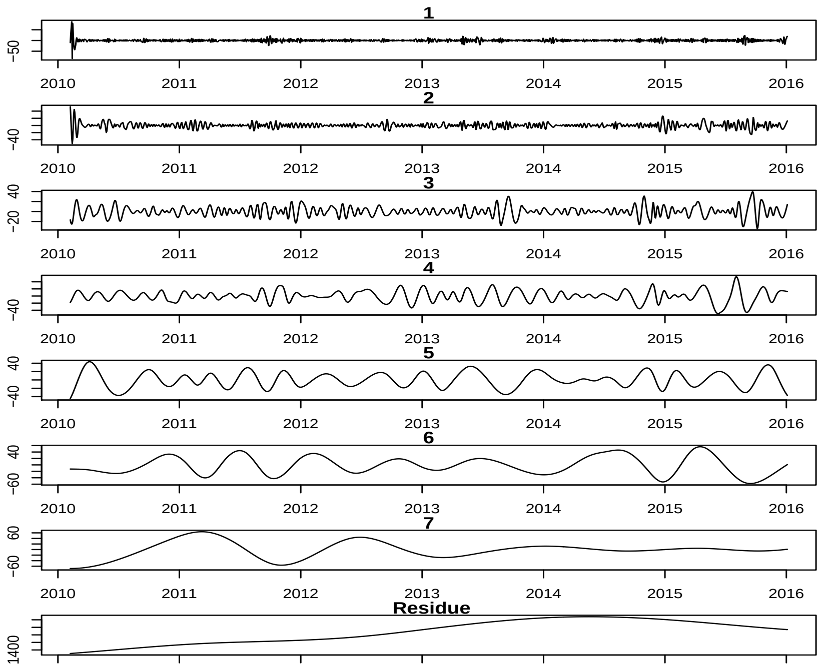

Figure 3: IMFs of KLSM

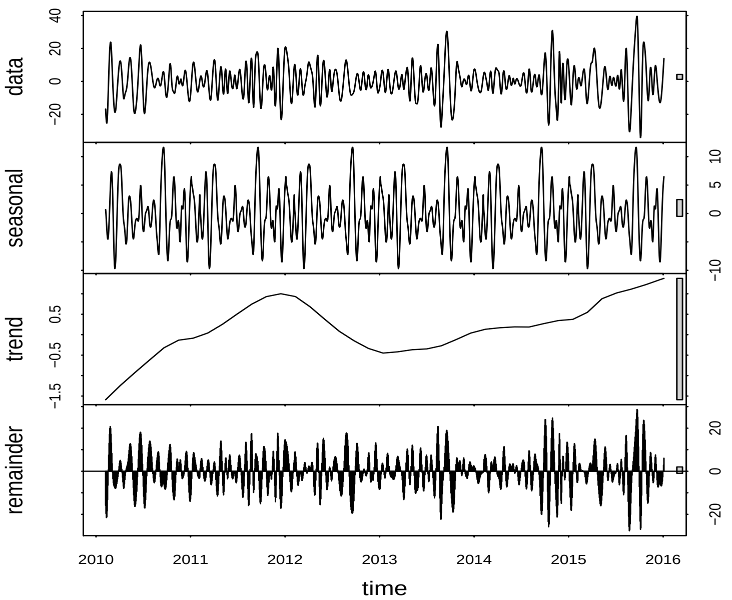

Figure 4: Results from the application of the STL decomposition on IMF(3) of KLSM.

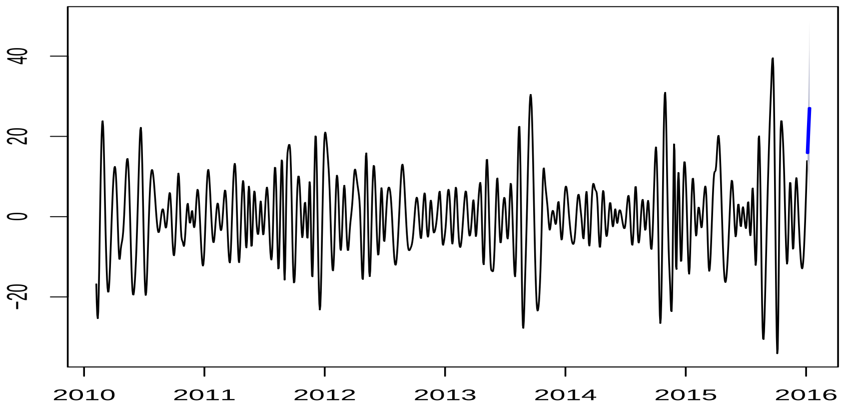

Figure 5: Forecast IMF(3) using EXP.

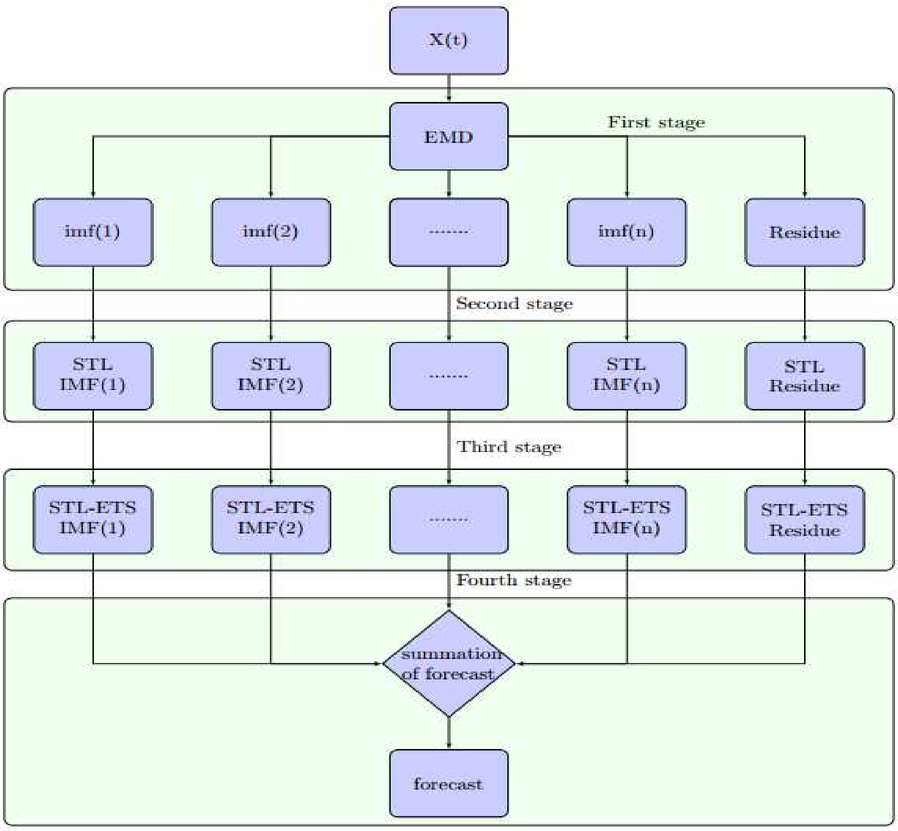

Figure 6: Flowchart of a hybrid EMD-EXP forecasting methodology

Algorithm 1 Overall EMD-EXP forecasting process

```
procedure EMD.EXP((time series,h))

    [IMF_1,...,IMF_n,residue] ← EMD(time series,"wave")

    for ts←each component of (IMFs and residue) do
        if (ts is seasonal) then
            ts.stl=[trend, seasonal, remainder] ← stl(ts )
        else
            seasonal ← 0
            ts.stl=[trend,0, remainder] ← loess(ts)
        end if

        h ahead forecast horizontal ← forecast(ETS(ts.stl), h)

    end for

    add the forecasting results together.
end procedure
```

4 Result and discussion

In this study, stock market time series data of 12 countries are used to present the forecasting accuracy of the EMD-EXP method. Seven forecasting methods are used in order to validate the forecasting performance of EMD-EXP. Table 3 shows three error measurements with their formula. Where $\widehat{y_i}$ is the forecast value of the variable y at time period i from knowledge of the actual series values. These measurements will be utilized to evaluate the forecasting accuracy for each method.

Table 3: Error measures are used in study

Name of measure error	Formula of measure error
Root Mean Squared Error	$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(y_i - \widehat{y_i})^2}$
Mean Absolute Error	$MAE = \frac{1}{n}\sum_{i=1}^{n}\lvert y_i - \widehat{y_i}\rvert$
Mean Absolute Percentage Error	$MAPE = \frac{1}{n}\sum_{i=1}^{n}\frac{\lvert y_i - \widehat{y_i}\rvert}{y_i}.100\%$

Selection of an error measurement has an important effect on the conclusions about which of a set of forecasting methods is most accurate. Table 4, Table 5 and Table 6 present the comparison result for EMD-EXP, when this comparison based on the mean of RMSE, MAE and MAPE for EMD-EXP and seven deferent forecasting methods for forecasting at $h = 2, 3, 4$ and 5 for each stock market time series data of 12 countries. While Figure 7 presents this comparison as well. From the result, the EMD-EXP method gives the smaller value of the mean error. This indicates that the forecast accuracy for EMD-EXP is better than the seven forecasting methods. As a conclusion, this proved that the hybrid method of EMD-EXP is giving a better forecasting result than the seven forecasting methods.

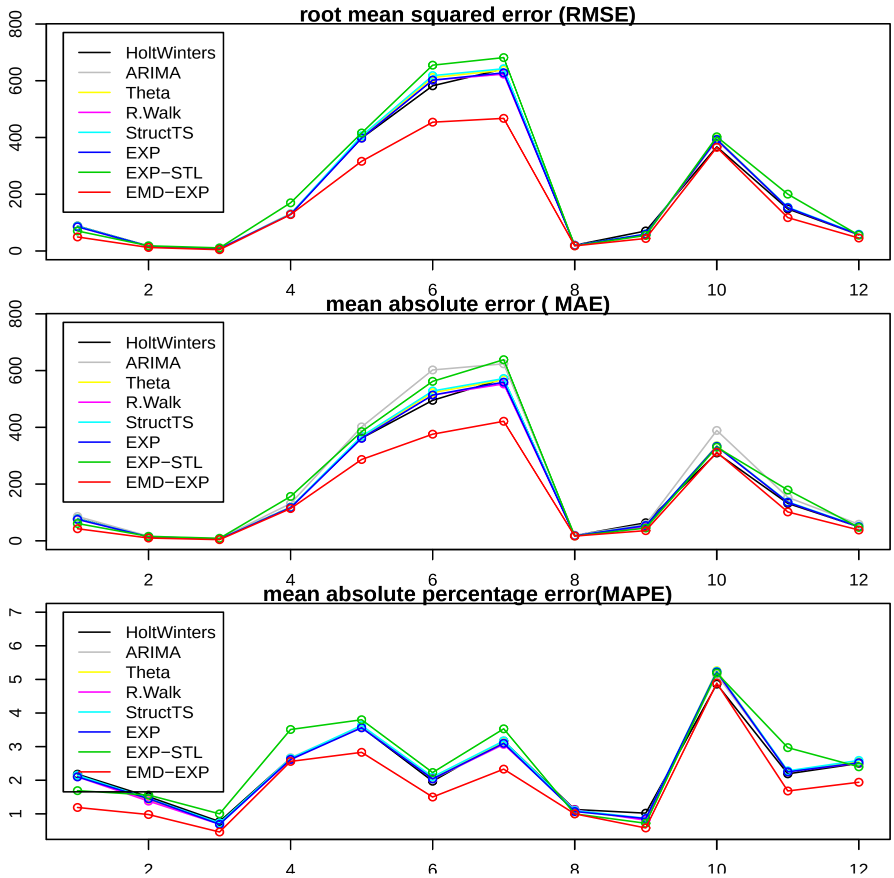

Figure 7: Presented the different mean errors which include RMSE, MAE and MAPE of EMD-EXP and seven forecasting methods for forecasting at $h = 2, 3, 4$ and 5 for each stock market time series data of 12 countries.

5 Conclusions

Time series forecasting still remains as one of the most difficult tasks due to the non-stationary and non-linearity of financial time series data. In this study, we have presented a new method, empirical mode decomposition with exponential smoothing method (EMD-EXP) for non-stationary and nonlinear time series forecasting. EMD-EXP based on Empirical Mode Decomposition (EMD), STL decomposition and exponential smoothing methods (EXP) have proposed. The daily stock market time series data of 12 countries are used to show the forecasting performance of the proposed EMD-EXP. Based on the three forecast accuracy measures, the results indicate that EMD-EXP forecasting performance is superior to seven traditional forecasting methods. Thus, this paper, has strengthened the idea that EMD-EXP forecasting method is suitable for non-stationary and nonlinear time series.

Table 4: The comparison of the mean of RMSE, MAE and MAPE for EMD-EXP and seven forecasting methods for forecasting at h = 2, 3, 4 and 5 for each stock market data of Belgium, Denmark, Estonia and France.

		Error measure		
Country	Method	RMSE	MAE	MAPE
Belgium	HoltWenter	88.18	78.1	2.18
	ARIMA	85.94	76.1	2.12
	Theta	86.28	76.39	2.13
	Random.Walk	85.25	75.48	2.1
	StructTS	87.27	77.26	2.15
	EXP	85.26	75.48	2.1
	EXP-STL	73.31	64.03	1.78
	EMD-EXP	49.4	42.62	1.19
Denmark	HoltWenter	17.54	14.96	1.51
	ARIMA	16.26	13.75	1.38
	Theta	16.62	14.08	1.42
	Random.Walk	16.24	13.72	1.38
	StructTS	17.07	14.51	1.46
	EXP	17.02	14.46	1.45
	EXP-STL	15.38	13.08	1.31
	EMD-EXP	12.31	9.82	0.98
Estonia	HoltWenter	8.27	6.8	0.77
	ARIMA	7.65	6.23	0.7
	Theta	7.69	6.27	0.71
	Random.Walk	7.5	6.08	0.69
	StructTS	7.91	6.46	0.73
	EXP	7.5	6.08	0.69
	EXP-STL	10.95	8.91	1
	EMD-EXP	4.58	4.13	0.46
France	HoltWenter	130.8	118.54	2.66
	ARIMA	131.39	119	2.67
	Theta	129.81	117.65	2.64
	Random.Walk	128.53	116.55	2.61
	StructTS	130.27	118.06	2.65
	EXP	128.73	116.7	2.62
	EXP-STL	169.83	156.42	3.51
	EMD-EXP	128.32	113.73	2.56

Table 5: The comparison of the mean of RMSE, MAE and MAPE for EMD-EXP and seven forecasting methods for forecasting at $h = 2$, 3, 4 and 5 for each stock market data of Germany, India, Japanese and Malaysia.

		Error measure		
Country	Method	RMSE	MAE	MAPE
Germany	HoltWenter	397.82	361.74	3.57
	ARIMA	401.52	366.24	3.61
	Theta	402.53	365.71	3.6
	Random.Walk	397.98	361.65	3.57
	StructTS	405.94	368.74	3.63
	EXP	397.97	361.62	3.56
	EXP-STL	415.89	384.87	3.8
	EMD-EXP	316.07	286.6	2.83
India	HoltWenter	582.3	495.16	1.97
	ARIMA	602.23	510.32	2.03
	Theta	611.47	522.19	2.08
	Random.Walk	602.09	513.59	2.04
	StructTS	618	528.16	2.1
	EXP	602.1	513.6	2.04
	EXP-STL	654.92	562	2.23
	EMD-EXP	454	375.79	1.5
Japanese	HoltWenter	641.46	570.91	3.17
	ARIMA	623.88	553.6	3.07
	Theta	637.43	567.44	3.15
	Random.Walk	623.41	552.98	3.07
	StructTS	642.56	572.05	3.17
	EXP	627.99	559.16	3.1
	EXP-STL	681.59	638.43	3.53
	EMD-EXP	467.45	421.35	2.33
Malaysia	HoltWenter	20.38	18.74	1.13
	ARIMA	19.62	18.08	1.09
	Theta	19.58	18.04	1.08
	Random.Walk	20.26	18.54	1.12
	StructTS	19.8	18.27	1.1
	EXP	19.34	17.8	1.07
	EXP-STL	18.28	16.61	1
	EMD-EXP	17.96	16.68	1

Table 6: The comparison of the mean of RMSE, MAE and MAPE for EMD-EXP and seven forecasting methods for forecasting at h = 2, 3, 4 and 5 for each stock market data of New Zealand, Saudi Arabia, United Kingdom and USA S&P500.

		Error measure		
Country	Method	RMSE	MAE	MAPE
New_Zealand	HoltWenter	70.59	63.67	1.02
	ARIMA	58.05	52.33	0.84
	Theta	57.87	52.46	0.84
	Random.Walk	55.68	50.48	0.81
	StructTS	59.88	54.22	0.87
	EXP	59.36	53.76	0.86
	EXP-STL	52.8	44.97	0.72
	EMD-EXP	44.11	35.9	0.58
Saudi Arabia	HoltWenter	367.46	309.62	4.86
	ARIMA	388.98	327.75	5.14
	Theta	396	335.56	5.26
	Random.Walk	392.84	332.7	5.22
	StructTS	394.03	333.77	5.24
	EXP	392.84	332.7	5.22
	EXP-STL	387.03	317.38	4.99
	EMD-EXP	365.23	312.17	4.89
United Kingdom	HoltWenter	148.2	131.8	2.19
	ARIMA	153.23	136.85	2.27
	Theta	153.62	136.76	2.27
	Random.Walk	152.5	135.74	2.25
	StructTS	154.16	137.26	2.28
	EXP	152.5	135.74	2.25
	EXP-STL	200.28	178.95	2.97
	EMD-EXP	117.58	101.77	1.68
USA_S.P500	HoltWenter	57.01	49.19	2.5
	ARIMA	58.42	50.21	2.55
	Theta	58.39	50.34	2.56
	Random.Walk	57.24	49.33	2.51
	StructTS	59.07	50.94	2.59
	EXP	57.46	49.53	2.52
	EXP-STL	59.37	50.28	2.56
	EMD-EXP	45.62	38.14	1.94

References

Abadan, S. and Shabri, A. (2014). Hybrid empirical mode decomposition-arima for forecasting price of rice. *Applied Mathematical Sciences*, 8(63):3133–3143.

Assimakopoulos, V. and Nikolopoulos, K. (2000). The theta model: a decomposition approach to forecasting. *International journal of forecasting*, 16(4):521–530.

Awajan, A. M. and Ismail, M. T. (2017). A hybrid approach emd-hw for short-term forecasting of daily stock market time series data. In *AIP Conference Proceedings*, volume 1870, page 060006. AIP Publishing.

Awajan, A. M., Ismail, M. T., and Al Wadi, S. (2017a). A hybrid emd-ma for forecasting stock market index. *Italian Journal of Pure and Applied Mathematics*, (38):313–332.

Awajan, A. M., Ismail, M. T., and Wadi, S. A. (2017b). Forecasting time series using emd-hw bagging. *International Journal of Statistics & Economics*, 18(3):9–21.

Chen, C.-F., Lai, M.-C., and Yeh, C.-C. (2012). Forecasting tourism demand based on empirical mode decomposition and neural network. *Knowledge-Based Systems*, 26:281–287.

Cheng, C.-H. and Wei, L.-Y. (2014). A novel time-series model based on empirical mode decomposition for forecasting taiex. *Economic Modelling*, 36:136–141.

Cleveland, R. B., Cleveland, W. S., McRae, J. E., and Terpenning, I. (1990). Stl: A seasonal-trend decomposition procedure based on loess. *Journal of Official Statistics*, 6(1):3–73.

Cleveland, W. S., Devlin, S. J., and Terpenning, I. J. (1981). The details of the sabl transformation, decomposition and calendar methods. *Bell Laboratories, Murray Hill, NJ.*

Cleveland, W. S., Grosse, E., and Shyu, W. M. (1992). Local regression models. *Statistical models in S*, pages 309–376.

Coughlin, K. and Tung, K.-K. (2004). 11-year solar cycle in the stratosphere extracted by the empirical mode decomposition method. *Advances in space research*, 34(2):323–329.

Dagum, E. B. (1988). The x11arima/88 seasonal adjustment method-foundations and users manual, time series research and analysis division. *Statistics Canada technical report.*

Deering, R. and Kaiser, J. F. (2005). The use of a masking signal to improve empirical mode decomposition. In *Acoustics, Speech, and Signal Processing, 2005. Proceedings.(ICASSP'05). IEEE International Conference on*, volume 4, pages iv–485. IEEE.

Gardner, E. S. (1985). Exponential smoothing: The state of the art. *Journal of forecasting*, 4(1):1–28.

Gardner, E. S. (2006). Exponential smoothing: The state of the artpart ii. *International journal of forecasting*, 22(4):637–666.

Guo, Z., Zhao, W., Lu, H., and Wang, J. (2012). Multi-step forecasting for wind speed using a modified emd-based artificial neural network model. *Renewable En-*

ergy, 37(1):241–249.

Harvey, A. C. and Peters, S. (1990). Estimation procedures for structural time series models. *Journal of Forecasting*, 9(2):89–108.

Holt, C. (1957). Forecasting trends and seasonals by exponentially weighted averages onr.

Huang, N. E., Shen, Z., Long, S. R., Wu, M. C., Shih, H. H., Zheng, Q., Yen, N.-C., Tung, C. C., and Liu, H. H. (1998). The empirical mode decomposition and the hilbert spectrum for nonlinear and non-stationary time series analysis. In *Proceedings of the Royal Society of London A: Mathematical, Physical and Engineering Sciences*, volume 454, pages 903–995. The Royal Society.

Huang, W., Sher, Y.-P., Peck, K., and Fung, Y. C. B. (2002). Matching gene activity with physiological functions. *Proceedings of the National Academy of Sciences*, 99(5):2603–2608.

Hyndman, R. J. and Athanasopoulos, G. (2014). *Forecasting: principles and practice.* OTexts.

Hyndman, R. J. and Billah, B. (2003). Unmasking the theta method. *International Journal of Forecasting*, 19(2):287–290.

Jaber, A. M., Ismail, M. T., and Altaher, A. M. (2013). Automatic boundary correction using empirical mode decomposition and local linear regression. *Far East Journal of Mathematical Sciences*, 2013:563.

Jaber, A. M., Ismail, M. T., and Altaher, A. M. (2014). Application of empirical mode decomposition with local linear quantile regression in financial time series forecasting. *The Scientific World Journal*, 2014.

Li, R. and Wang, Y. (2008). Short-term wind speed forecasting for wind farm based on empirical mode decomposition. In *Electrical Machines and Systems, 2008. ICEMS 2008. International Conference on*, pages 2521–2525. IEEE.

Lin, C.-S., Chiu, S.-H., and Lin, T.-Y. (2012). Empirical mode decomposition–based least squares support vector regression for foreign exchange rate forecasting. *Economic Modelling*, 29(6):2583–2590.

Lu, Y., Oruklu, E., and Saniie, J. (2013). Chirplet signal and empirical mode decompositions of ultrasonic signals for echo detection and estimation. *Journal of Signal and Information Processing*, 4(02):149.

Makridakis, S. and Hibon, M. (2000). The m3-competition: results, conclusions and implications. *International journal of forecasting*, 16(4):451–476.

Muck, J. and Skrzypczynski, P. (2012). Can we beat the random walk in forecasting cee exchange rates? *Available at SSRN 2163518.*

Oh, H.-S., Suh, J.-H., and Kim, D.-H. (2009). A multi-resolution approach to non-stationary financial time series using the hilbert-huang transform. *Korean Journal of Applied Statistics*, 22(3):499–513.

Pegels, C. C. (1969). Exponential forecasting: Some new variations. *Management Science*, pages 311–315.

Peiris, M. and Perera, B. (1988). On prediction with fractionally differenced arima models. *Journal of Time Series Analysis*, 9(3):215–220.

Persons, W. M. (1919). *Indices of General Business Condoitions*. Harvard Univ. Committee on Economic research.

Rilling, G., Flandrin, P., Goncalves, P., et al. (2003). On empirical mode decomposition and its algorithms. In *IEEE-EURASIP workshop on nonlinear signal and image processing*, volume 3, pages 8–11. NSIP-03, Grado (I).

Rossi, M. and Brunelli, D. (2015). Forecasting data centers power consumption with the holt-winters method. In *Environmental, Energy and Structural Monitoring Systems (EESMS), 2015 IEEE Workshop on*, pages 210–214. IEEE.

Rowland, P. et al. (2003). *Forecasting the USD/COP Exchange Rate: A Random Walk with a Variable Drift*. Banco de la República.

Tatinati, S. and Veluvolu, K. C. (2013). A hybrid approach for short-term forecasting of wind speed. *The Scientific World Journal*, 2013.

Theodosiou, M. (2011). Forecasting monthly and quarterly time series using stl decomposition. *International Journal of Forecasting*, 27(4):1178–1195.

Turner, L. W. and Witt, S. F. (2001). Forecasting tourism using univariate and multivariate structural time series models. *Tourism Economics*, 7(2):135–147.

Winters, P. R. (1960). Forecasting sales by exponentially weighted moving averages. *Management Science*, 6(3):324–342.

Yang, D., Sharma, V., Ye, Z., Lim, L. I., Zhao, L., and Aryaputera, A. W. (2015). Forecasting of global horizontal irradiance by exponential smoothing, using decompositions. *Energy*, 81:111–119.

Yu, L., Wang, S., and Lai, K. K. (2008). Forecasting crude oil price with an emd-based neural network ensemble learning paradigm. *Energy Economics*, 30(5):2623–2635.

Zhang, R. R., Ma, S., Safak, E., and Hartzell, S. (2003). Hilbert-huang transform analysis of dynamic and earthquake motion recordings. *Journal of Engineering Mechanics*, 129(8):861–875.