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A queuing model for dealing with patients with severe disease

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This paper suggests a proposed single server queueing model for severe diseases especially in Outpatient Department. The Outpatient Department of a hospital is visited by patients of all types of disease. Some of these diseases require immediate medical attention as severe complications may arise if treatment is delayed. The goal of the study was to develop a queueing model considering patients with severe disease and to study the improvement in the service time using the model. The single server queueing model was modified and analyzed. The efficiency of the model was tested by using outpatient medical service, arrivals and departure of patients over a period of one year of a local hospital in Guwahati. The result indicated the average outpatient medical service response times for service improve over the general model.

keywords: Queuing Theory, Single server queueing model, disease, hospital, patients.

1 Introduction

It is known that when entities, typically referred to as customers, arrive at a service facility requesting service and cannot be served immediately upon arrival a queue is formed. In healthcare delivery systems, patients are typically the customers and the outpatient department is the foremost point of service facility.

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Outpatient department is one of the most important parts of hospital management and is visited by large section of community. This is the first point of contact between patient and hospital staff. The problems faced by the patients in that department are overcrowding, delay in consultation, lack of proper guidance etc. which lead to patients becoming dissatisfied. Every patient in each hospital is in search of hassle free and quick services in the fast growing world which is only possible with optimum utility of the resources through multitasking in a single server system in the outpatient department for better services.

Queues are everywhere, particularly in healthcare delivery systems. At the same time, it has many negative impacts because delay in receiving needed services causes different types of loss, viz. time, economic and status of health, which further triggers worsening of their medical conditions that can increase subsequent treatment costs and poor health outcomes. This is more so when the patient is suffering from some severe disease, that needs immediate medical attention.

The transient solution of queueing problems are associated with varying arrival and departure rates. Saaty (1961) has considered the transient single server model and derived the queue size of the distribution. Collings and Stoneman (1976) have considered the transient $M/M/\infty$ model and derived the queue size distribution for a time dependent arrival and departure rates in the form of probability generating function.

Parthasarathy and Sharafali (1989) has also derived a simple method for the time dependent solution for a number of patients in the system in the multi server model. Abol'nikov (1968) derives the generating function of the queue size distribution at any time. Shanbhag (1966) considers the general service time distribution and shows that if the departure rate is one at point of time and the initial queue size is empty,then the resulting queue size distribution is always Poisson with a time dependent parameter. Clarke (1956) studied queues resulting from non homogeneous Poisson processes and provided a complete theoretical solution to the single server model.

Leese and Boyd (1966), provide a useful discussion of the numerical method that have been proposed. Bagchi and Templeton (1972) also applied the queues resulting from non homogeneous Poisson processes but their applicable to the more general forms or method is not time depending. Mohit and Sharma (2010) considered the $M/M/\infty$ model and obtained the probabilities and expected queue length of system for a time dependent values of arrival and service rates.

Palvannan and Teow (2012) established the queueing theory and that helps to quantify the appropriate service capacity to meet the patient demand, balancing system utilization and the patient's wait time. Xie et al. (2014) also derived the closed formula to evaluate the response time performance by assuming exponential response time, and investigate the system-theoretic properties. Saghafian et al. (2015) considered the emergency department patient flow by using optimization method. Dhar and Mahanta

(2014) compared the single server as well as multiple servers queuing model in an out patient department.

Our study is based on the finding from an outpatient department in a public hospital.The daily arrival pattern of the number of patients are large and that has limited time period during the hospital hours to serve patients visited from outpatient department especially those patient suffering from severe disease. Here we studied single server queueing model of the waiting time in queue and in the system in case of patient who suffer from severe disease.

2 Model Description

The single server queueing model is the simplest model which was first derived by Kendall and it is also known as Kendall's notation. Here we shall assume that the arrival of patients in a fixed time interval belongs to Poisson probability distribution at an average rate of patients per unit time. It is also assumed that the service time was exponentially distributed, with an average rate of patients per unit of time. Also assume that the service discipline is FIFO, no restriction on wating space and the infinite number of patients. The basic structure and its generalization have been discussed in queueing literature.

2.1 Assumption of the model

Following assumptions are made throughout this study for the purpose of the single server of outpatient patient department.

- The arrival of a patients at any given time is independent of the arrival of a patient at any other time.
- Service time and inter-arrival time are independent of each other.
- The number of patients are infinite.
- server is single.

2.2 Notation

The basic notation are given here

- λ = Arrival Rate,
- μ = Service Rate,
- X_1 = Number of arrival of patients who suffer from severe disease (say s),
- Y = Total number of patients who arrived,

- X_2 = Number of patients suffering from severe disease and getting service,
- Z = Total number of patients getting service,
- p_1 = Proportion of severe patients arriving in a hospital and
- p_2 = Proportion of severe patients getting service in a hospital.

Some of the results stated below, shall be used in the proposed single server queueing model.
Now, the arrival rate of the customers and the rate of service vary with time.

Result 1. If $X_1|Y \sim Bin(y, p_1)$ and $Y \sim Poi(\lambda)$, then $X_1 \sim Poi(\lambda p_1)$.

Proof: In a hospital, number of patients arrive, each patient suffer from a particular disease (say S).

$$\begin{aligned} P(X_1 = x) &= \sum_{y=0}^{\infty} P(X_1 = x, Y = y) \\ &= \sum_{y=0}^{\infty} P(X_1 = x|Y = y).P(Y = y) \\ &= \frac{e^{-\lambda p_1}(\lambda p_1)^x}{x!} \end{aligned}$$

Result 2. If $X_2|Z \sim Bin(z, p_2)$ and $Z \sim Poi(\mu)$, then $X_2 \sim Poi(\mu p_2)$.

Proof: As Stated in Result 1.

Result 3. If $X_1 \sim Poi(\lambda p_1)$ and $X_2 \sim Poi(\mu p_2)$, then $S = X_1 + X_2 \sim Poi(\lambda p_1 + \mu p_2)$.

Proof:

$$\begin{aligned} P(X_1 + X_2 = S) &= \sum_{x=0}^{s} P(X_1 = x)P(X_2 = s - x) \\ &= e^{-(\lambda p_1 + \mu p_2)} \sum_{x=0}^{s} \binom{s}{x} (\lambda p_1)^x (\mu p_2)^{s-x} \\ &= \frac{e^{-(\lambda p_1 + \mu p_2)}}{s!} (\lambda p_1 + \mu p_2)^s \end{aligned}$$

Result 4. If $X_1 \sim Poi(\lambda p_1)$ and $S = X_1 + X_2 \sim Poi(\lambda p_1 + \mu p_2)$ then $X_1|S \sim Bin(x_1, \pi_1)$, Where $\pi_1 = \frac{\lambda p_1}{\lambda p_1 + \mu p_2}$.

Proof:

$$\begin{aligned} P(X_1 = x|S = s) &= P(X_1 = x|X_1 + X_2 = s) \\ &= \binom{s}{x}(\pi_1)^x(1-\pi_1)^{s-x} \end{aligned}$$

Result 5. If $X_2 \sim Poi(\mu p_2)$ and $S = X_1 + X_2 \sim Poi(\lambda p_1 + \mu p_2)$, then $X_2|S \sim Bin(x_2, \pi_2)$, where $\pi_2 = \frac{\mu p_2}{\lambda p_1 + \mu p_2}$.

Proof: As stated in result 4.

2.3 Single Server Queueing model vs proposed model:

Let

- A = number of arrival of patients who suffer from a particular disease and which follows Poisson distribution with rate λ ;
- S = number of patients who suffer from disease and getting service in the hospital which follows Poisson distribution with rate $\lambda p_1 + \mu p_2$;
- SE = number of patients getting service which follows Poisson distribution with rate μ.

From the general model,

$P_n(t+h) = P_{n-1}(t)\ [P(A = 1, SE = 0)] + P_{n+1}(t)\ [P(A = 0, SE = 1)] + P_n(t)\ [P(A = 0, SE = 0)]$

Let us consider the patients who suffer from a particular disease say S in the hospital.Then,

$$\begin{aligned} P_n(t+h) &= P_{n-1}(t\ )[P(A = 1, S = 1, SE = 0) + P(A = 1, S = 0, SE = 0)] \\ &\quad + P_{n+1}(t)\ [P(A = 0, SE = 1, S = 0) + P(A = 0, SE = 1, S = 1)] \\ &\quad + P_n(t)\ [P(A = 0, SE = 0)] \end{aligned}$$

Let $A_1 = (A = 1 \cap S = 1)$, $A_2 = (A = 1 \cap S = 0)$,

$S_1 = (SE = 0 \cap S = 1)$ and $S_2 = (SE = 1 \cap S = 0)$

Also let $A_1 \cap SE = \phi$, $A_2 \cap SE = \phi$, $S_1 \cap A = \phi$ and $S_2 \cap A = \phi$

Therefore, $P_n(t+h) = I_{n-1}(t) + I_{n+1}(t) + I_n(t)$

$$\begin{aligned} I_{n-1}(t) &= P_{n-1}(t)\ [P(A_1)\ P(SE) + P(A_2)\ P(SE)] \\ &= P_{n-1}(t)\ P(SE)\ [P(A_1) + P(A_2)] \\ &= P_{n-1}(t)\ P(SE)\ P(A=1)[P(S=1|A=1) + P(S=0|A=1)] \\ &= P_{n-1}(t)\ P(SE=0)\ P(A=1)[P_{11} + P_{01}] \\ &= P_{n-1}(t)\ (\lambda h)\ (1-\mu h) \sum_{i=0}^{1} P_{i1} \end{aligned}$$

where $P_{i1} = P(S=i|A=1)$ for all $i = 0, 1$

$$\begin{aligned} I_{n+1}(t) &= P_{n+1}(t)\ [P(S_1)\ P(A) + P(S_2)\ P(A)] \\ &= P_{n+1}(t)P(A)\ [P(S_1) + P(S_2)] \\ &= P_{n+1}(t)P(A)\ P(SE=1)[P(S=1|SE=1) + P(S=0|SE=1)] \\ &= P_{n+1}(t)\ P(SE=1)P(A=0)[B_{11} + B_{01}] \\ &= P_{n+1}(t)\ (\mu h)\ (1-\lambda h) \sum_{j=0}^{1} B_{j1} \end{aligned}$$

where $B_{j1} = P(S=j|SE=1)$ for all $j = 0, 1$

$$\begin{aligned} I_n(t) &= P_n(t)\ P(A=0, SE=0) \\ &= P_n(t)\ P(A=0)\ P(SE=0) \\ &= P_n(t)(1-\lambda h)\ (1-\mu h) \end{aligned}$$

$$\begin{aligned} P_n(t+h) &= P_{n-1}(t)\ (\lambda h)\ (1-\mu h) \sum_{i=0}^{1} P_{i1} + P_{n+1}(t)\ (\mu h)\ (1-\lambda h) \sum_{j=0}^{1} B_{j1} \\ &\quad + P_n(t)(1-\lambda h)\ (1-\mu h) \end{aligned}$$

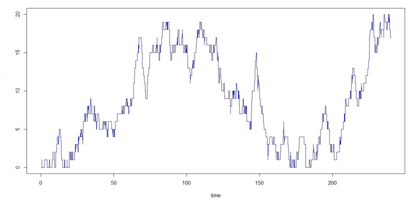
3 A Case Study on Model validation from original patients record

We demonstrate our methods using two real outpatient datasets:
1) the patients waiting time;
2) the service time.
The data has been collected directly from public hospital by using the direct observational method. These data contains all the relevant information regarding each patient like patients arrival time, patients departure time and suffered from disease or diseases

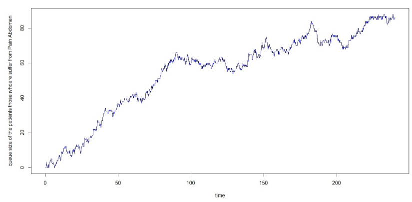
respectively.

In particular, we apply the simple single server queuing model or M/M/1 queue to model the number of patients who suffered from severe disease.

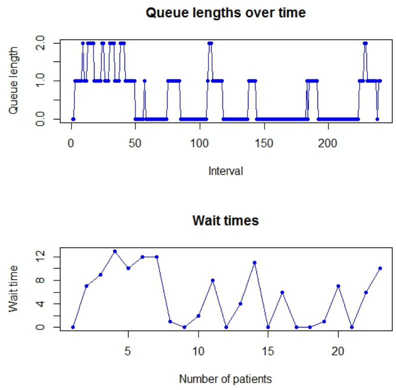
3.1 Results and Discussion



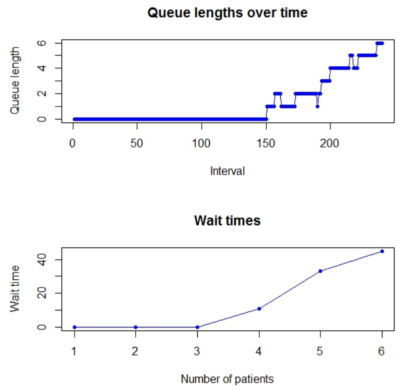
(a) Arrival pattern of the patients during the period



(b) Arrival pattern of the patients who suffered from pain in abdomen



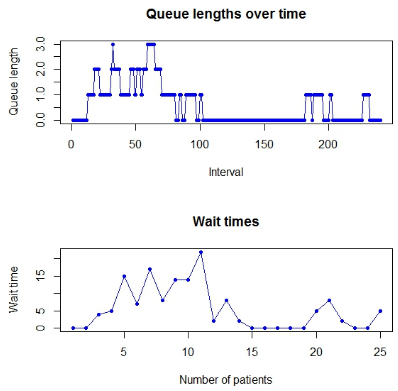
(c) Queue length and waiting time of the patients who suffered from abdomen pain in single server model



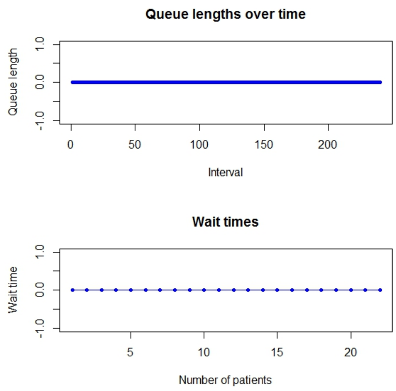
(d) Queue length and waiting time of the patients who suffered from abdomen pain in proposed single server model

Figure (a) represents the daily arrival pattern of the patients who suffered from severe disease. Here we also show that the maximum number of patients visiting the hospital is from around 10 a.m. in the morning and for a duration of 50 to 150 minutes. After that the arrival pattern of the patients fall.

Figure (b) shows the arrival pattern of the patients who suffered from pain in abdomen. Here we considered the pain in abdomen cases out of the daily arrival pattern of the patients who suffered from severe disease. From the data, we have got many more severe diseases but we consider only Pain in Abdomen because of Appendix or Appendicitis (Appendicitis is inflammation of the Appendix). As at that situation the patient can



(e) Queue length and waiting time of the patients who suffered from abdomen pain in simulated single server model



(f) Queue length and waiting time of the patients who suffered from abdomen pain in simulated proposed single server model

not wait for service, we apply the proposed single server queueing model.
Figure (c) and (d) displays the queueing length of the patients who suffered from abdomen pain in single server model as well as proposed single server model. In this figure, we show that the original model contains much variation of the waiting time of the patients as compared to the proposed model. The figure (e) and (f) simply suggest that the arrival pattern of the patients increase as compared to the original single server queuing model. We also examined the arrival pattern of the number of patients during this period. It is clearly demonstrated that the proposed single server queueing model is more general and can also be used as a proof-of-concept or a smoothing tool for the standard rate of service time.

Conclusion

In this paper, a simple single server queueing model of patients who suffered from severe disease or diseases has been put forward along with the waiting time in queue and patients waiting time in system. Setting the waiting time parameter to zero, these reduce to the well-known waiting time distribution of the single server queueing model. The figures representing a primary data and a simulated one have shown that the proposed single server queueing model is better as compared to the original single server queueing model.

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